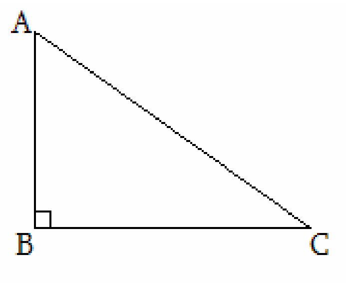
LEE COUNTY INVITATIONAL

Geometry Individual Test

January 26, 2008

"NOTA" means None Of These Answers. Figures are NOT drawn to scale. For the purpose of this test, a kite is a quadrilateral with two pairs of disjoint consecutive congruent sides.

1) In $\triangle ABC$, $AB = 3$, $BC = 4$, and $\angle B = 90$. Find the length of $\overline{AC}$.



a) $2\sqrt{3}$

b) $\frac{5}{2}$

c) 25

d) 5

e) NOTA

2) At 4:30 PM, a statue of an Ohio State Buckeye, which is 6 feet tall, casts a shadow that is $13\frac{1}{2}$ feet long. If it is instantaneously torn down and replaced by a statue of a Florida Gator that is 10 feet tall, how long will its shadow be?

a) $4\frac{4}{9}$ *ft.*

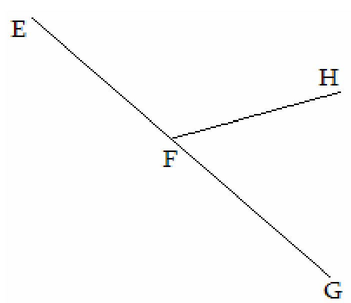
b) $22\frac{1}{2}$ *ft.*

c) $10\frac{3}{4}$ *ft.*

d) 25 *ft.*

e) NOTA

3) EFG is a straight line, and $\angle GFH \neq 90°$. If $\angle EFH$ is the vertex angle of an isosceles triangle, then what is the measure of each base angle?



a) $\frac{1}{2}\angle HFG$

b) $180° - \angle EFH$

c) $90° - \angle EFH$

d) $\frac{1}{2}\angle EFH$

e) NOTA

4) On the Cartesian plane, the set of points that are equidistant from two fixed, distinct points form a ___________.

a) line
b) segment
c) ray
d) midpoint
e) NOTA

5) How many of the following are NOT a way to prove right triangle congruency in Euclidean Geometry, where H is the hypotenuse, L is a leg, and A is an angle?

HL	HA	LA
HH	AA	LL

a) 0
b) 1
c) 2
d) 3
e) NOTA

6) The measures of six consecutive exterior angles of a heptagon are in the ratio 1:2:3:4:5:6. The remaining exterior angle is $66°$. Find the measure of the smallest interior angle.

a) $77\frac{1}{7}°$
b) $96°$
c) $114°$
d) $152°$
e) NOTA

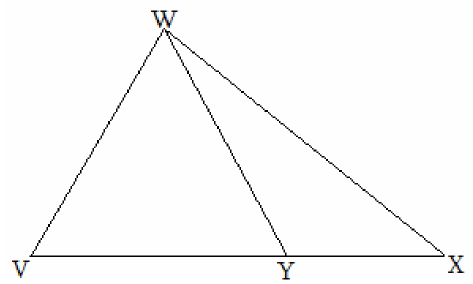
7) Find the number of sides of a regular n-gon whose number of diagonals is equal to $n^2 - 4n - 7$.

a) 2
b) 5
c) 7
d) 12
e) NOTA

8) When proving triangle congruency, which of the following is the expanded form of CPCTC?

a) Congruent Parts of Corresponding Triangles are Congruent
b) Congruent Parts of Composite Triangles are Congruent
c) Corresponding Parts of Congruent Triangles are Congruent
d) Corresponding Parts of Corresponding Triangles are Congruent
e) NOTA

9) Given ΔVWX, with Y on $\overline{VX}$, find the length of $\overline{WY}$ if $VY = 3$, $VW = 3$, $VX = 4$, and $WX = 5$.



a) 3
b) $3\sqrt{2}$
c) 2
d) $2\sqrt{2}$
e) NOTA

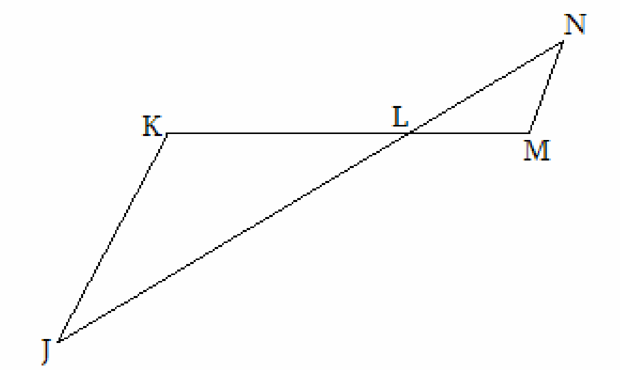
10) What is the inverse of the contrapositive of the inverse of the converse of the contrapositive of the inverse of $\sim p \to q$?

a) $\sim q \to p$
b) $p \to \sim q$
c) $\sim p \to q$
d) $q \to \sim p$
e) NOTA

11) ABCDEFGHIJKLMN is a regular 14-gon. Starting at A, diagonal $\overline{AF}$ is drawn, then $\overline{FK}$, then $\overline{KB}$, etc. When this cycle is complete, what fraction of the total number of possible diagonals is drawn?

a) $\frac{1}{11}$
b) $\frac{2}{11}$
c) $\frac{3}{11}$
d) $\frac{4}{11}$
e) NOTA

12) In the figure to the right, $\angle J \cong \angle N$, $LK = 3$, $LM = 2$, and $MN = 3$. Find the length of $\overline{JK}$.



a) 6
b) 5.5
c) 5
d) 4.5
e) NOTA

13) Given $\triangle HUG$, with $HU = 2\sqrt{3}$, $UG = 6$, and $GH = 4\sqrt{3}$, find the value of $(\sin H)^2 + (\cos H)^2$.

a) $\frac{1}{2}$
b) 1
c) $\frac{3}{2}$
d) 2
e) NOTA

14) The lengths of the sides of a non-degenerate triangle are 6, 8, and x, where x is a positive integer. The lengths of the sides of another non-degenerate triangle are 36, 64, and x^2. How many values of x exist?

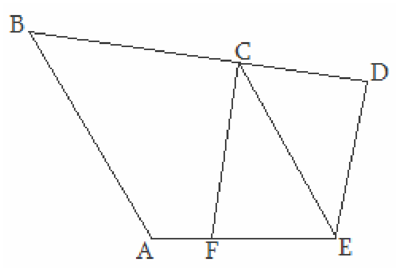
a) 5
b) 7
c) 9
d) 11
e) NOTA

15) According to the Laws of Quadrilaterals, which of the following *must* be true?

I. All squares are rectangles.
II. All rhombi are rectangles.
III. Some rhombi are equiangular.
IV. Some parallelograms are squares.

a) I, III, IV only
b) I, II, IV only
c) I, II, III only
d) I, IV only
e) NOTA

16) In quadrilateral ABDE, C is on $\overline{BD}$, F is on $\overline{AE}$, $AE \perp DE$, $CE = CD$, and $\overrightarrow{CF}$ bisects $\angle BCE$. If $m\angle ABC = 51°$ and $m\angle CDE = 40°$, find $m\angle FAB$.



a) 109°
b) 129°
c) 171°
d) 179°
e) NOTA

17) Given: $\triangle ICY$ and $\triangle HOT$ with $\overline{IC} \cong \overline{HO}$, $\overline{IY} \cong \overline{HT}$, and $\angle I \cong \angle H$. Which method could be used to prove that $\triangle ICY \cong \triangle HOT$?

a) SAS
c) SSS
c) HL
d) ASA
e) NOTA

18) How many distinct equilateral triangles can be drawn such that the triangle shares at least two vertices with a regular heptagon?

a) 14
b) 21
c) 28
d) 42
e) NOTA

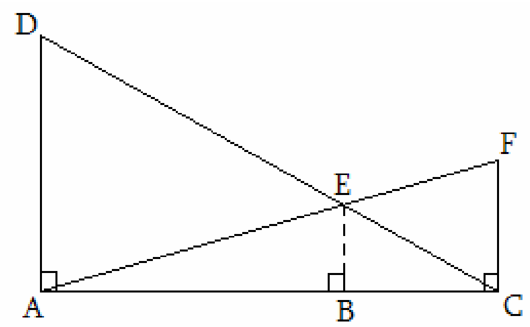
19) The Greek letter Σ (sigma) is commonly used to indicate a sum in the following fashion: the number on the bottom is the starting number, the number on top is the ending number, and whatever lies after the sigma is what needs to be summed. For example,

$$\sum_{n=2}^{8}\left(n^2\right)=2^2+3^2+4^2+5^2+6^2+7^2+8^2=203$$

What is $\sum_{n=3}^{6} f(x)$, where $f(x)=$ the measure of one exterior angle in a regular n-gon?

a) 720 b) 360 c) 378
d) 342 e) NOTA

20) In the figure to the right, $\overline{FC}$ and $\overline{DA}$ are perpendicular to $\overline{AC}$, $AD=10$, and $FC=5$. Find the length of $\overline{EB}$.

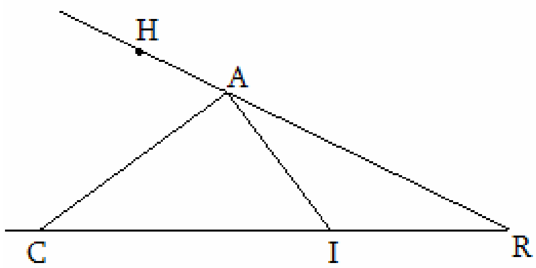


a) $\frac{5}{2}$
b) $\frac{10}{3}$
c) $\frac{7}{2}$
d) 4
e) NOTA

21) Who was the man born around 640 BC in Miletus who brought geometry to Greece from Egypt and is considered by some to be *The Father of Modern Geometry*?

a) Euclid b) Pythagoras c) Thales
d) Archimedes e) NOTA

22) In the figure to the right, $\angle RAI \cong \angle IAC$, $AR=8$, $IR=5$, and $AC=4$. Find the length of $\overline{IC}$.



a) 5 b) $\frac{5}{2}$
b) 10 e) NOTA
e) $\frac{15}{2}$

23) From a point in the interior of a regular hexagon, perpendiculars drawn to the sides of the hexagon have lengths, in size order, of 14, 18, 19, 23, 24, and 28. What is the length of a side of this hexagon?

a) 21
b) $7\sqrt{3}$
c) $10\sqrt{3}$
d) $14\sqrt{3}$
e) NOTA

24) Given scalene, acute $\triangle GAP$, where $m\angle GAP = 56°$ and $m\angle APG = x°$, for how many *integral* values does x exist?

a) 53
b) 52
c) 37
d) 38
e) NOTA

25) What is the ratio between the sum of the squares of the side lengths to the sum of the squares of the median lengths of any triangle?

a) Cannot be determined
b) $3:2$
c) $4:3$
d) $4:1$
e) NOTA

26) Given the supplement of the complement of the supplement of $\angle GUM$, which of the following are true about $\angle GUM$?

I. $90° < m\angle GUM < 180°$
II. If $m\angle GUM = 95°$, then the supplement of the complement of the supplement of $m\angle GUM = 95°$
III. If $\angle GUM$ was the vertex angle of obtuse, isosceles triangle $\triangle MUG$, then the supplement of the complement of the supplement of $\angle GUM = \frac{1}{2}(180 - \angle UGM)$.

a) II, III only
b) I, II only
c) I, III only
d) I, II, III
e) NOTA

27) Fill in the blank: _________ is the term for the point of intersection of skew lines in Euclidean Geometry.

a) Equilibrium
b) Node
c) Vanishing Point
d) No such point exists
e) NOTA

28) Which of the following can be the intersection of two planes?

I. Point
II. Line
III. Plane

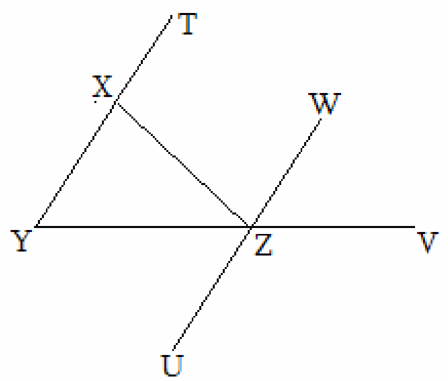
a) I, II, and III
b) I, II only
c) II, III only
d) I, III only
e) NOTA

29) How many triples in the box below are Pythagorean Triples?

- 3, 4, 5
- $3n,\ 4n,\ 5n$ for all positive values of n
- $3^n,\ 4^n,\ 5^n$ for all positive values of n
- $3+n,\ 4+n,\ 5+n$ for all positive values of n
- $3-n,\ 4-n,\ 5-n$ for all positive values of n
- $n^3,\ n^4,\ n^5$ for all positive values of n

a) 2
b) 3
c) 4
d) 5
e) NOTA

30) In the figure to the right, $m\angle XZY = 40°$, $m\angle UZV = 115°$, and $\overline{TXY} \parallel \overline{WZU}$, with $\overline{YZV}$ as a transversal. Find $m\angle ZYX + m\angle WZV$.

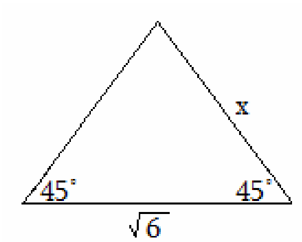


a) 130°
b) 105°
c) 80°
d) 50°
e) NOTA

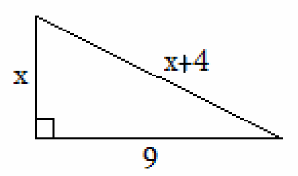
Question 1

For each question, find the value of x.

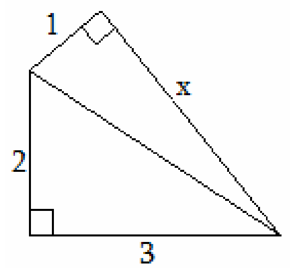
A)



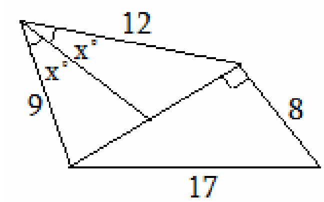
B)



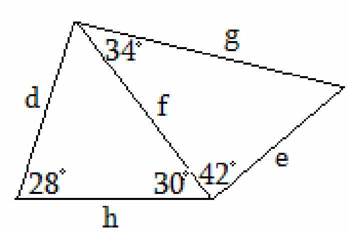
C)



D)



Question 2



A) Given the quadrilateral above, find the longest side.
B) Given the quadrilateral above, find the second longest side.
C) Given the quadrilateral above, find the second shortest side.
D) Given the quadrilateral above, find the shortest side.

Question 3

In $\triangle ABC$, $D, E,$ and F are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively. $AB = \frac{3}{2}x + 2y$, $BC = 4x - 2$, $AC = 2x - 2z$, $DE = -3x + 3y$, $EF = y + 3z$, and $DF = x + 3$.

A) What is the value of x?
B) What is the value of y?
C) What is the value of z?
D) What is the perimeter of $\triangle DEF$?

Question 4

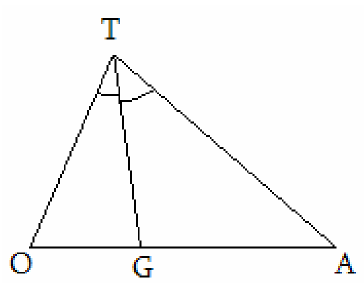
A) The equation of the line through (3, -4) and (6, 11) is written in the form $Ax + By = C$, where A, B, and C are relatively prime integers, and A is positive. Find the value of $A + B + C$.
B) The equation of the line through (-4, 5) and parallel to 3x+6y=17 is written in the form $Dx + Ey = F$, where D, E, and F are relatively prime integers, and D is positive. Find the value of $D + E + F$.
C) The equation of the line through (2, -7) and perpendicular to the line 2x-4y=7 is written in the form $Gx + Hy = I$, where G, H, and I are relatively prime integers, and G is positive. Find the value of $G + H + I$.
D) The equation of the perpendicular bisector of $\overline{JK}$, with $J(1,6)$ and $K(0,20)$ is written in the form $Lx + My = N$, where L, M, and N are relatively prime integers, and L is positive. Find the value of $L + M + N$.

Question 5

A) Find the perimeter of a rhombus given diagonals of length $2\sqrt{5}$ and $\sqrt{7}$.
B) Find the length of the longer side of a rectangle with a diagonal of length $\sqrt{5}$ and a perimeter of 6.
C) Find the perimeter of an isosceles trapezoid with median of length 10, base angle measures of $30°$, and a height of $3\sqrt{3}$.
D) Find the perimeter of regular hexagon PQRSTU given that the perimeter of $\triangle PRT$ is 3.

Question 6

In $\triangle OAT$, the angle bisector of $\angle T$ meets $\overline{OA}$ at G, $OG = \frac{10}{3}$, $GA = \frac{26}{3}$, and $OT = 5$.



A) What is the length of $\overline{AT}$?
B) What is the measure of $\angle GOT$?
C) What is the value of $\cos(\angle OAT)$ expressed as a fraction in simplest form?
D) What is the length of $\overline{GT}$?

Question 7

On the Cartesian plane, $\triangle ABC$ is an equilateral triangle with coordinates $A(0,0)$, $B(6,0)$, and $C(3,3\sqrt{3})$. The line $y = x\sqrt{3} - 1$ intersects $\triangle ABC$ at points $D(q,r)$ and $E(s,t)$, where $q < s$ and $r < t$.

A) Find the shortest distance between $\overline{AC}$ and $\overline{DE}$.

B) Find $m\overline{DE}$ in the form $\dfrac{X}{Y}$.

C) Find $q + r + s + t$.

D) Find the ratio, in fractional form, between the perimeter of triangle BED and the perimeter of triangle ABC.

Question 8

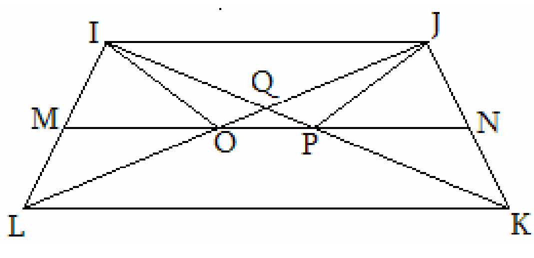
Given isosceles trapezoid IJKL with median $\overline{MN}$ and diagonals $\overline{JL}$ and $\overline{IK}$, $\overline{IJ} \parallel \overline{LK}$, $\overline{IJ} = 6$, $\overline{LK} = 16$, a height of $5\sqrt{3}$, and O and P are on $\overline{MN}$.

A) Find the length of $\overline{MO}$.

B) Find the length of $\overline{OP}$.

C) Find the length of $\overline{JP}$.

D) Find the length of $\overline{PK}$.



Question 9

A) The supplement of the complement of $\angle A$ is equal to seven times the measure of $\angle A$. Find the measure of $\angle A$.

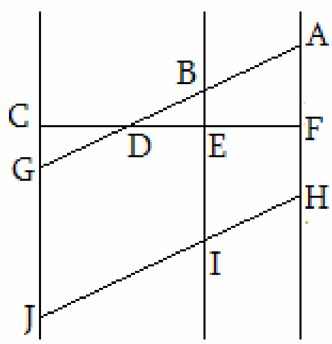
B) The angle measure of $\angle B$ is n°, and the angle measure of the complement of $\angle B$ is $(36n-21)^\circ$. Find the value of n.

C) $m\angle CDE = m\angle FDG = 40^\circ$, and E, D, and F are collinear. Find the sum of all possible values of $m\angle CDG$.

D) The angle measures of a right triangle are x°, $(x+y)^\circ$, and $(x+5y)^\circ$ where x and y are both positive integers. Find the value of $\frac{x}{y}$.

Question 10

In the figure to the right, $\overline{CJ} \parallel \overline{BI} \parallel \overline{AH}$, $\overline{AG} \parallel \overline{JH}$, $\overline{CF} \perp \overline{AH}$, $\angle BDE = 30^\circ$, $BE = 3$, and $AF = 4$.



A) Find $m\angle EIJ$.

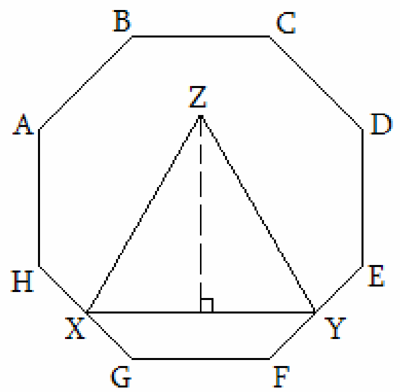
B) Find the length of $\overline{DA}$.

C) Find the length of $\overline{HI}$.

D) Find the sum of the angle measures in pentagon DEIJG.

Question 11

Given regular octagon ABCDEFGH with perimeter of 64, where X is the midpoint of $\overline{GH}$, Y is the midpoint of $\overline{EF}$, and $\triangle XYZ$ is an equilateral triangle with Z lying in the interior of octagon ABCDEFGH.



A) Find $m\overline{XY}$.
B) Find the shortest distance between the altitude drawn from Z to side $\overline{XY}$ of $\triangle XYZ$, and $\overline{DE}$.
C) Find the shortest distance between $\overline{XY}$ and $\overline{FG}$.
D) Find the ratio, in fractional form, of the perimeter of pentagon GXZYF to the perimeter of octagon ABCDEFGH.

Question 12

In the Geometric Zoo, where all animals are extremely geometrically-inclined, a group of animals decided to play a game where they have to use a geometric sentence to guess the measure of an angle. Josef the Jaguar thinks the measure of the angle is equal to the number of diagonals in a dodecagon. Pamela the Penguin thinks the measure of the angle is equivalent to the degree measure of the sum of 5 exterior angles of a regular 36-gon. Alexis the Armadillo thinks the measure of the angle is equal to the perimeter of an equilateral triangle with an altitude of length $10\sqrt{3}$. Gina the Giraffe thinks the measure of the angle is equal to the number of degrees in the last remaining angle of a quadrilateral with angle measures of 113°, 102°, and 88°. The angle is measured with a protractor, and it was observed that one animal was exactly correct, two animals were three degrees off, and one animal was seven degrees off.

A) Who was correct in guessing the measure of the angle?
B) Who were three degrees off in guessing the measure of the angle?
C) Who was seven degrees off in guessing the measure of the angle?
D) What was the measure of the angle?

Question 13

Please classify the following statements as either TRUE or FALSE. (Note: The words TRUE or FALSE must be submitted. No points will be given to any single letter responses.)

A) Co-planar lines that do not intersect must be parallel.
B) The angle measures of vertical angles must be equivalent.
C) A proof by induction must begin with a contradiction to the statement trying to be proven.
D) If the side lengths of two similar triangles are in the ratio 1:2, then the ratio of their perimeters must be 1:6.

Question 14

Square	Rhombus	Trapezoid
Rectangle	Quadrilateral	Parallelogram

Of the figures that appear in the box above, how many of them can be described by the explanations below?

A) Must have perpendicular diagonals.
B) Must have bisecting diagonals.
C) Must have at least one pair of parallel sides.
D) Can have all sides of differing lengths.

Question 15

Given $\triangle ABC$, with $AB = 6$, $BC = 8$, $AC = 4$, and $\triangle DEF$, with $DE = 3 \cdot AB$, $EF = 2 \cdot BC$, $DF = 4 \cdot AC$,

A) Classify $\triangle ABC$ as equilateral, isosceles, or scalene.
B) Classify $\triangle ABC$ as acute, right, or obtuse.
C) Classify $\triangle DEF$ as equilateral, isosceles, or scalene.
D) Classify $\triangle DEF$ as acute, right, or obtuse.

1) D -- Pythagorean theorem!

$x^2 = 3^2 + 4^2$

$x = 5$

2) B – Similar Triangles

$\frac{6}{13.5} = \frac{10}{13.5 + x}$

$81 + 6x = 135$

$6x = 54$

$x = 9$

$Shadow = 22.5$

3) A – The two base angles, when added together, will be equal to the measure of the exterior angle of the vertex angle. So, the measure of one base angle is equal to one half of the measure of the exterior angle.

4) A – A line defines the set of points equidistant from two points.

5) C – HH and AA do not prove right triangle congruency. (HH can't even be real)

6) B – The remaining exterior angle is 66 degrees, and the sum of the exterior angles in any heptagon is 360 (as it is with any polygon). So,

$360 = x + 2x + 3x + 4x + 5x + 6x + 66$

$360 = 21x + 66$

$294 = 21x$

$14 = x$

The smallest interior angle will be adjacent to the largest exterior angle, which has an angle measure of 6(14)=84. So, the smallest interior angle has a length of 180-84=96.

7) C – The number of diagonals in a polygon is $\frac{n(n-3)}{2}$. So,

$\frac{n(n-3)}{2} = n^2 - 4n - 7$

$n^2 - 3n = 2n^2 - 8n - 14$

$0 = n^2 - 5n - 14$

$(n-7)(n+2) = 0$

$n = 7$

8) C – Corresponding Parts of Congruent Triangles are Congruent

9) B – Given that $WX = 5$, $VX = 4$, and $VW = 3$, $\triangle VWX$ is a right triangle! So,
$$(WY)^2 = 3^2 + 3^2$$
$$WY = 3\sqrt{2}$$

10) A – In the statement "*The inverse of the contrapositive of the inverse of the converse of the contrapositive of the inverse*," you have 3 inverses, which cancel each other to 1 inverse. You also have 2 contrapositives, which cancel each other out to nothing, as well as 1 converse. 1 inverse and 1 converse become 1 contrapositive. So, $\sim p \to q$ becomes $\sim q \to \sim\sim p$ or $\sim q \to p$.

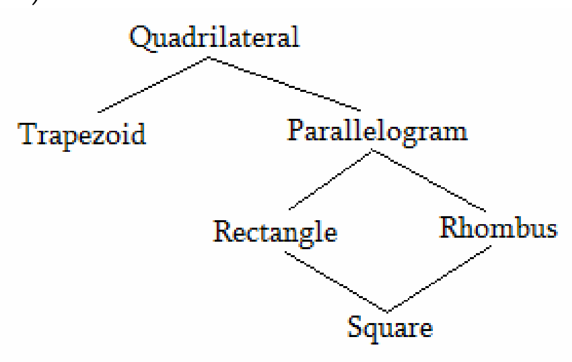
11) B – The diagonals drawn will be the segments AF, FK, KB, BG, GL, LC, CH, HM, MD, DI, IN, NE, EJ, JA, and then it begins again with segment AF. This method creates 14 diagonals. The total number of diagonals is $\frac{14(11)}{2} = 77$. So the fraction drawn is $\frac{14}{77} = \frac{2}{11}$.

12) D --
$$\frac{LK}{LM} = \frac{JK}{MN}$$
$$\frac{3}{2} = \frac{JK}{3}$$
$$JK = \frac{9}{2} = 4.5$$

13) B --
$$\sin H = \frac{UG}{GH}$$
$$\cos H = \frac{HU}{GH}$$
$$\sin^2 H + \cos^2 H = \frac{(UG)^2 + (HU)^2}{(GH)^2} = \frac{(GH)^2}{(GH)^2} = 1$$
This is actually true for all values of H!

14) E --
For the first condition, the possible integral values of x fall under $8-6<x<8+6$ or $2<x<14$. These values are 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13.
For the second condition, the possible integral values of x fall under $64-36<x^2<64+36$ or $28<x^2<100$. These values are 6, 7, 8, and 9. So, there are 4 values.

15) A --



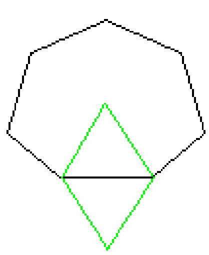
I. True
II. False
III. True (would be a square)
IV. True

16) D --

$\angle FAB = 360-90-51-40=179$

17) A --
Since you are given two sides and the included angle, SAS would be appropriate here.

18) D --
There are going to be two on each diagonal, as well as two on each side. An example of two are shown to the right. So, there are 42 triangles.



19) D --
$$\sum_{n=3}^{6}\left(\frac{360}{n}\right)=\frac{360}{3}+\frac{360}{4}+\frac{360}{5}+\frac{360}{6}=120+90+72+60=342$$

20) B –

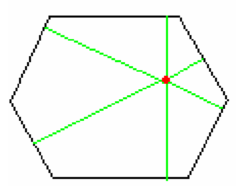
$$x = \frac{ab}{a+b} = \frac{10 \cdot 5}{10+5} = \frac{50}{15} = \frac{10}{3}$$

21) C --
The first important geometer mentioned in history is Thales of Miletus, a Greek who lived about 600 BC. Thales is credited with several simple but important theorems, including the proof that an angle inscribed in a semicircle is a right angle.

22) B --
$\overrightarrow{AI}$ is the angle bisector of $\angle CAR$, so $\frac{AC}{AR} = \frac{IC}{IR}$. So,

$$\frac{4}{8} = \frac{x}{5}$$
$$8x = 20$$
$$2x = 5$$
$$x = \frac{5}{2}$$

23) D --
Given any point in the interior of a hexagon and drawing perpendiculars to all sides, if you take the sum of these lengths, you get the value of six times the length of the apothem, since the lengths pair up to become collinear. A picture is shown to the right. So, the sum of the lengths is 126, which means the apothem has a length of 21. This means that $\frac{1}{2}s = \frac{21}{\sqrt{3}} = 7\sqrt{3}$ so $s = 14\sqrt{3}$.



24) B --
Since the triangle must be acute, the following conditions must hold true.
$$56 + x > 90 \rightarrow x > 34$$
$$56 + x < 180 \rightarrow x < 124$$
$$x < 90$$
So, the value of x must be between 34 and 90, exclusive. This leaves 55 possible values for x. However, the triangle must also be scalene, so the following conditions must also hold true.
$$56 \neq x$$
$$56 \neq 180 - 56 - x \rightarrow x \neq 68$$
$$x \neq 180 - 56 - x \rightarrow x \neq 62$$
So, this eliminates 3 possible values for x, leaving 52 possible values.
25) C --

The sum of the squares of the medians is

$$\left(\sqrt{\frac{a^2}{2}+\frac{b^2}{2}-\frac{c^2}{4}}\right)^2+\left(\sqrt{\frac{a^2}{2}+\frac{c^2}{2}-\frac{b^2}{4}}\right)^2+\left(\sqrt{\frac{c^2}{2}+\frac{b^2}{2}-\frac{a^2}{4}}\right)^2$$

, which looks daunting, until it simplifies…

$$\frac{a^2}{2}+\frac{b^2}{2}-\frac{c^2}{4}+\frac{a^2}{2}+\frac{c^2}{2}-\frac{b^2}{4}+\frac{c^2}{2}+\frac{b^2}{2}-\frac{a^2}{4}$$

$$\frac{2a^2}{4}+\frac{2b^2}{4}-\frac{c^2}{4}+\frac{2a^2}{4}+\frac{2c^2}{4}-\frac{b^2}{4}+\frac{2c^2}{4}+\frac{2b^2}{4}-\frac{a^2}{4}$$

$$\frac{3a^2}{4}+\frac{3b^2}{4}+\frac{3c^2}{4}$$

$$\frac{3}{4}\left(a^2+b^2+c^2\right)$$

So, the sum of the squares of the medians of the triangle is ¾ the sum of the square of the sides. So, the ratio is 4:3.

26) E --

I. TRUE. The supplement of the complement of the supplement will be equal to the supplement of the angle. However, since the complement can be taken, the supplement of the original angle MUST be acute. So the original angle MUST be obtuse.

II. FALSE. $180-(90-(180-95))=175$.

III. FALSE.

$$\angle GUM = x$$

$$\angle UGM = \frac{180-\angle GUM}{2}$$

$$180-(90-(180-\angle GUM)) = 270-\angle GUM$$

$$\frac{1}{2}(180-\angle UGM) = \frac{1}{2}\left(180-\frac{180-\angle GUM}{2}\right)$$

$$\frac{1}{2}(180-\angle UGM) = \frac{180+\angle GUM}{4} \neq 270-\angle GUM$$

27) D --

Skew lines never intersect!

28) C --

Planes cannot intersect at a point, since they go on forever. Most planes intersect at a line, and planes that coincide intersect at a plane.

29) A --

$3^2 + 4^2 = 5^2$

$(3n)^2 + (4n)^2 = (5n)^2$

$3^{2n} + 4^{2n} \neq 5^{2n}$

$(3+n)^2 + (4+n)^2 \neq (5+n)^2$

$(3-n)^2 + (4-n)^2 \neq (5-n)^2$

$n^6 + n^8 \neq n^{10}$

30) A --

$\angle YZU \cong \angle WZV$, since they are vertical angles. Since $\overline{TXY} \parallel \overline{WZU}$, $\angle ZYX \cong \angle WZV$. Since $m\angle UZV = 115$, $m\angle YZU = 115$. So, $m\angle ZYX + m\angle WZV = 65 + 65 = 130$.

LEE COUNTY INVITATIONAL

Geometry Answers

January 26, 2008

Individual Answers

1. D
2. B
3. A
4. A
5. C
6. B
7. C
8. C
9. B
10. A
11. B
12. D
13. B
14. E
15. A
16. D
17. A
18. D
19. D
20. B
21. C
22. B
23. D
24. B
25. C
26. E
27. D
28. C
29. A
30. A

Team Answers

1. A) $\sqrt{3}$
   B) $\frac{65}{8}$ or $8\frac{1}{8}$ or 8.125
   C) $2\sqrt{3}$
   D) 45

2. A) h
   B) d
   C) g
   D) e

3. A) 4
   B) 5
   C) 1
   D) 18

4. A) 23
   B) 9
   C) 0
   D) -389

5. A) $6\sqrt{3}$
   B) 2
   C) $20+12\sqrt{3}$
   D) $2\sqrt{3}$

6. A) 13
   B) 90
   C) $\frac{12}{13}$
   D) $\frac{5\sqrt{13}}{3}$

LEE COUNTY INVITATIONAL

Geometry Answers

January 26, 2008

7. A) $\frac{1}{2}$

   B) $\frac{18-\sqrt{3}}{3}$ *or* $6-\frac{\sqrt{3}}{3}$

   C) $\frac{5+7\sqrt{3}}{2}$

   D) $\frac{18-\sqrt{3}}{18}$ *or* $1-\frac{\sqrt{3}}{18}$

8. A) 3

   B) 5

   C) $\sqrt{19}$

   D) 7

9. A) 15

   B) 3

   C) 360

   D) 4

10. A) 120

    B) 8

    C) 2

    D) 540

11. A) $8+4\sqrt{2}$

    B) $4+4\sqrt{2}$

    C) $2\sqrt{2}$

    D) $\frac{4+\sqrt{2}}{8}$

12. A) Gina the Giraffe

    B) Alexis the Armadillo and Josef the Jaguar

    C) Pamela the Penguin

    D) 57

13. A) true

    B) true

    C) false

    D) false

14. A) 2

    B) 4

    C) 5

    D) 2

15. A) scalene

    B) obtuse

    C) isosceles

    D) acute

SOUTHWEST FLORIDA INVITATIONAL

Geometry Team Solutions

January 26, 2008

1) A.

$$x^2 + x^2 = \left(\sqrt{6}\right)^2$$
$$2x^2 = 6$$
$$x = \sqrt{3}$$

B.

$$x^2 + 9^2 = (x+4)^2$$
$$x^2 + 81 = x^2 + 8x + 16$$
$$8x = 65$$
$$x = \frac{65}{8} = 8\frac{1}{8} = 8.125$$

C.

$$1^2 + x^2 = \left(\sqrt{3^2+2^2}\right)^2$$
$$1 + x^2 = 13$$
$$x^2 = 12$$
$$x = 2\sqrt{3}$$

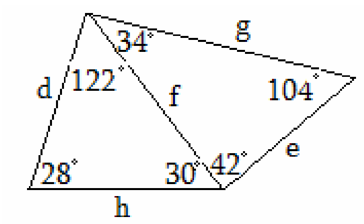
D.

$$8^2 + a^2 = 17^2$$
$$a = 15$$

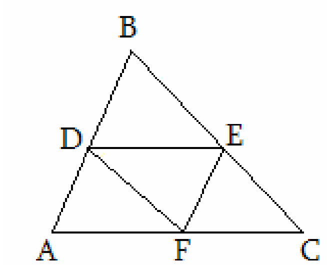
9-12-15 is a right triangle, so x=45.

2) The smallest side of the triangle on the left is the largest side of the triangle on the right. So, from largest to smallest, the sides go h, d, f, g, e.

A. h
B. d
C. g
D. e



3) $\overline{AB} = 2\overline{EF}$, $\overline{BC} = 2\overline{DF}$, $\overline{AC} = 2\overline{DE}$



A.

$$\overline{BC} = 2\overline{DF}$$
$$4x - 2 = 2(x+3)$$
$$2x = 8$$
$$x = 4$$

SOUTHWEST FLORIDA INVITATIONAL

Geometry Team Solutions

January 26, 2008

B&C.

$AB = 2EF$

$\frac{3}{2}x + 2y = 2(y + 3z)$

$\frac{3}{2}(4) + 2y = 2y + 6z$

$6 = 6z$

$z = 1$

$AC = 2DE$

$2x - 2z = 2(-3x + 3y)$

$2x - 2z = -6x + 6y$

$2(4) - 2(1) = -6(4) + 6y$

$8 - 2 = -24 + 6y$

$6y = 30$

$y = 5$

D.

$P = -3x + 3y + y + 3z + x + 3$

$P = -2x + 4y + 3z + 3$

$P = -8 + 20 + 3 + 3$

$P = 18$

4) A.

$slope = \frac{11 - (-4)}{6 - 3} = 5$

$5(x - 3) = y - (-4)$

$5x - 15 = y + 4$

$5x - y = 19$

$5 - 1 + 19 = 23$

B.

$3x + 6y = 17$

$6y = -3x + 17$

$y = \frac{-1}{2}x + \frac{17}{6}$

$slope = \frac{-1}{2}$

$\frac{-1}{2}(x - (-4)) = y - 5$

$x + 4 = -2(y - 5)$

$x + 4 = -2y + 10$

$x + 2y = 6$

$1 + 2 + 6 = 9$

SOUTHWEST FLORIDA INVITATIONAL

Geometry Team Solutions

January 26, 2008

C.

$$2x-4y=7$$
$$4y=2x-7$$
$$\frac{1}{2}x-\frac{7}{4}=y$$
$$slope=\frac{1}{2}$$
$$PerpSlope=-2$$
$$-2(x-2)=y-(-7)$$
$$-2x+4=y+7$$
$$-2x-y=3$$
$$2x+y=-3$$
$$2+1-3=0$$

D.

$$Slope=\frac{20-6}{0-1}=-14$$
$$PerpSlope=\frac{1}{14}$$
$$Midpt=\left(\frac{1+0}{2},\frac{6+20}{2}\right)=\left(\frac{1}{2},13\right)$$
$$\frac{1}{14}\left(x-\frac{1}{2}\right)=y-13$$
$$x-\frac{1}{2}=14y-182$$
$$2x-1=28y-364$$
$$2x-28y=-363$$
$$2-28-363=-389$$

5) A.

$$\left(\frac{2\sqrt{5}}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2=s^2$$

$$5+\frac{7}{4}=s^2$$

$$\frac{27}{4}=s^2$$

$$s=\frac{3\sqrt{3}}{2}$$

$$P=4s=6\sqrt{3}$$

B.

$P=6,$ so $semiperimeter=3$

$Length=L$, $Width=3-L$

$$L^2+(3-L)^2=\left(\sqrt{5}\right)^2$$

$$2L^2-6L+4=0$$

$$L^2-3L+2=0$$

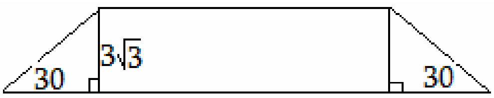
$$(L-2)(L-1)=0$$

$$L=2,1$$

The length is 2 and the width is 1, so the length of the longer side is 2.

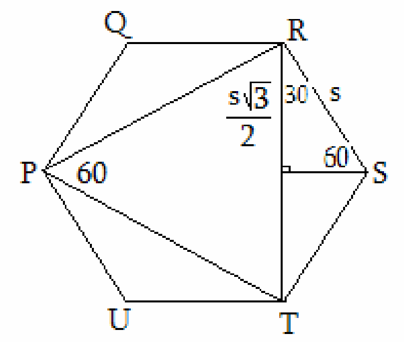
C.

Both side triangles are 30-60-90 triangles, where the side across from the 30 degree angle measure is $3\sqrt{3}$, which means the leg is $6\sqrt{3}.$ The sum of the bases is two times the length of the median, or 20. So the perimeter is $20+12\sqrt{3}.$



D.

Each side of triangle PRT is going to be $\sqrt{3}$ times the length of the side of hexagon PQRSTU. So the ratio of the perimeter of the triangle to the perimeter of the hexagon is $3\sqrt{3}:6$, or $\sqrt{3}:2$.



$$\frac{\sqrt{3}}{3}=\frac{2}{P_{HEX}}$$

$$P_{HEX}=2\sqrt{3}$$

SOUTHWEST FLORIDA INVITATIONAL

Geometry Team Solutions

January 26, 2008

6) A.

$$\frac{OT}{OG} = \frac{AT}{AG}$$

$$\frac{5}{10/3} = \frac{AT}{26/3}$$

$$AT = 13$$

B.

With sides of length 5, 12, and 13, triangle OAT is a right triangle! $\angle GOT = 90^\circ$

C.

$$\cos(\angle OAT) = \frac{adjacent}{hypotenuse} = \frac{12}{13}$$

D.

$$5^2 + \left(\frac{10}{3}\right)^2 = (GT)^2$$

$$25 + \frac{100}{9} = (GT)^2$$

$$\frac{325}{9} = (GT)^2$$

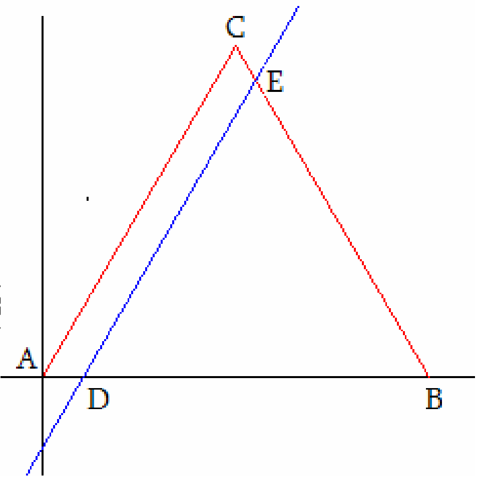
$$GT = \frac{5\sqrt{13}}{3}$$

A.

shortest distance is $\left|\frac{(\sqrt{3})(0)+(-1)(0)-1}{\sqrt{(\sqrt{3})^2+(-1)^2}}\right| = \frac{1}{2}$

B.

$DE = DB = AB - AD = \frac{18-\sqrt{3}}{3}$ sinceAB=6 and AD= $\frac{\sqrt{3}}{3}$



C.

$$D\left(\frac{\sqrt{3}}{3},0\right).$$

E is the vertex at the height of triangle DEB. So,

$$s = xvalue_{Midpt-\overline{DB}} = \frac{6-\frac{\sqrt{3}}{3}}{2} = \frac{18-\sqrt{3}}{6} + \frac{\sqrt{3}}{3} = \frac{18+\sqrt{3}}{6}$$

$$t = h_{\triangle DEB} = side \cdot \sqrt{3} = \frac{\sqrt{3}\left(18-\sqrt{3}\right)}{6} = \frac{18\sqrt{3}-3}{6}$$

$$q+r+s+t = \frac{\sqrt{3}}{3} + 0 + \frac{18+\sqrt{3}}{6} + \frac{18\sqrt{3}-3}{6}$$

$$q+r+s+t = \frac{2\sqrt{3}+18+\sqrt{3}+18\sqrt{3}-3}{6}$$

$$q+r+s+t = \frac{15+21\sqrt{3}}{6} = \frac{5+7\sqrt{3}}{2}$$

D.

$$\frac{Perimeter\left(\triangle BED\right)}{Perimeter\left(\triangle ABC\right)} = \frac{3\left(\frac{18-\sqrt{3}}{3}\right)}{3\left(6\right)}$$

$$\frac{Perimeter\left(\triangle BED\right)}{Perimeter\left(\triangle ABC\right)} = \frac{18-\sqrt{3}}{18}$$

8) A.

MO is half of the length of segment IJ in triangle IJL, since MO is parallel to IJ and because MO lies on the median. So, MO=3.

B.

Since the trapezoid is isosceles, MO=PN=3. MN, the median, has a length of 11. So,

$$OP = MN - MO - PN$$

$$OP = 11 - 6$$

$$OP = 5$$

C.

Draw an auxiliary line perpendicular to the lower base from the top base through P. This imaginary line hits the upper base at A, and the lower base at B.

$$JA = \frac{IJ - OP}{2} = \frac{1}{2}$$

$$(JA)^2 + (AP)^2 = (JP)^2$$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2 = (JP)^2$$

$$\frac{1}{4} + \frac{75}{4} = (JP)^2$$

$$(JP)^2 = 19$$

$$JP = \sqrt{19}$$

D.

$$KB = \frac{LK - OP}{2} = \frac{11}{2}$$

$$(KB)^2 + (BP)^2 = (PK)^2$$

$$\left(\frac{11}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2 = (PK)^2$$

$$(PK)^2 = \frac{121}{4} + \frac{75}{4}$$

$$(PK)^2 = \frac{196}{4}$$

$$PK = 7$$

9)

A.

$$180 - (90 - x) = 7x$$

$$90 + x = 7x$$

$$90 = 6x$$

$$x = 15$$

SOUTHWEST FLORIDA INVITATIONAL

Geometry Team Solutions

January 26, 2008

B.

$n+36n-21=90$

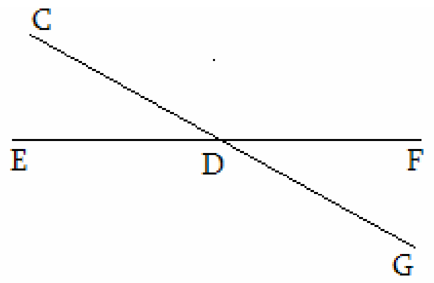
$37n-21=90$

$37n=111$

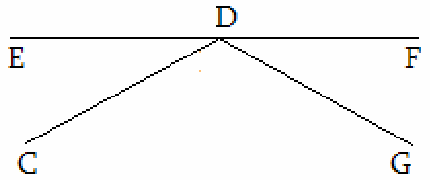
$n=3$

C.

There are three possible scenarios, and all are shown below.



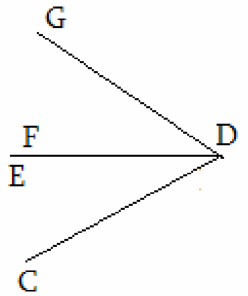
$m\angle CDG=180$



$m\angle CDG=180-m\angle CDE-m\angle FDG$

$m\angle CDG=180-40-40$

$m\angle CDG=100$



$m\angle CDG=m\angle CDE+m\angle FDG$

$m\angle CDG=40+40$

$m\angle CDG=80$

100+180+80=360

D.

The angles of a triangle add up to 180 degrees, so $x+(x+y)+(x+5y)=180$ or $3x+6y=180$. Also, the largest angle in the triangle is 90 degrees. Since x and y are both positive integers, then the angle with measure $(x+5y)^\circ$ will be the largest. So, $x+5y=90$.

So, it's a system of equations.

$3x+6y=180$

$x+5y=90 \rightarrow x=90-5y$

$3(90-5y)+6y=180$

$270-15y+6y=180$

$9y=90$

$y=10$

$x+5(10)=90$

$x+50=90$

$x=40$

$\frac{x}{y}=\frac{40}{10}=4$

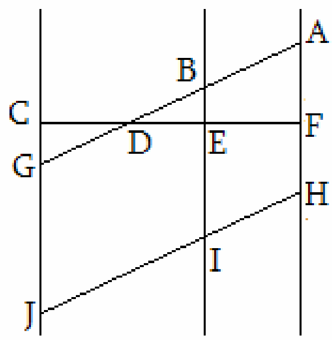
10) A.

$m\angle BDE=30$

$m\angle DBE=60$

$m\angle EIH=60$

$m\angle EIJ=180-\angle EIH=120$



B.

$\frac{BE}{AF}=\frac{DB}{DA}$

$DB=2\cdot BE \qquad (30,60,90)$

$\frac{3}{4}=\frac{6}{DA}$

$DA=8$

C.

$HI=AB=DA-DB$

$HI=8-6=2$

D.

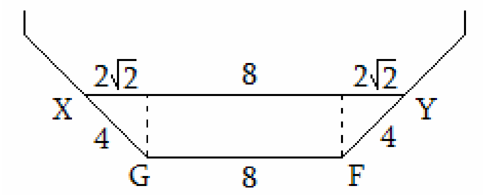
The sum of the angles in any pentagon is $180(5-2)=540$

11) A.

Draw imaginary altitudes on trapezoid XYFG stemming from F and G. These have a length of $2\sqrt{2}$, since it creates a 45-45-90 triangle with the side of the octagon. This makes the length of $\overline{XY}=8+4\sqrt{2}$

B.

This length is equal



to the apothem of the octagon, which is

$$d=\frac{1}{2}HE$$

$$\frac{FG+HE}{2}=XY$$

$$\frac{8+HE}{2}=8+4\sqrt{2}$$

$$8+HE+8\sqrt{2}$$

$$HE=8+8\sqrt{2}$$

$$d=\frac{1}{2}HE=4+4\sqrt{2}$$

C.

This length is equal to the length of the altitude of trapezoid XYFG = $2\sqrt{2}$

D.

$$\frac{Perimeter_{GXZYF}}{Perimeter_{ABCDEFGH}}=\frac{4+8+4\sqrt{2}+8+4\sqrt{2}+4+8}{64}$$

$$\frac{Perimeter_{GXZYF}}{Perimeter_{ABCDEFGH}}=\frac{32+8\sqrt{2}}{64}$$

$$\frac{Perimeter_{GXZYF}}{Perimeter_{ABCDEFGH}}=\frac{4+\sqrt{2}}{8}$$

SOUTHWEST FLORIDA INVITATIONAL

Geometry Team Solutions

January 26, 2008

12) Josef the Jaguar --

$$D = \frac{12(12-3)}{2}$$

$$D = 54$$

Pamela the Penguin

$$Ext = \frac{360}{36} = 10^\circ$$

$$5 \cdot Ext = 50$$

Alexis the Armadillo

$$s = \frac{2 \cdot Alt}{\sqrt{3}} = \frac{2 \cdot 10\sqrt{3}}{\sqrt{3}} = 20$$

$$P = 3 \cdot 20 = 60$$

Gina the Giraffe

$$A = 360 - (113 + 102 + 88)$$

$$A = 360 - 303 = 57$$

Use trial and error to figure out the measure of the angle. Since one of the guesses was exactly right, there are only four possible correct answers. Two of the guesses were exactly three degrees off, so they are either the same measure, or six degrees apart with the correct guess three degrees away from each. There are no guesses that are exactly the same, so they must be six degrees apart. These two angles are 54 and 60, which means 57 was exactly correct. This holds true since the remaining angle measure, 50, is seven degrees off.

A) Gina the Giraffe
B) Alexis the Armadillo and Josef the Jaguar
C) Pamela the Penguin
D) 57

13) A.
TRUE – since they are coplanar lines. If they weren't coplanar, they could be skew.
B.
TRUE – vertical angles are always equivalent
C.
FALSE – this is true of an indirect proof.

D.
FALSE – sides of ratio a:b have perimeters of length a:b

14) A.
Squares and Rhombi must have perpendicular diagonals. 2.
B.
All parallelograms have bisecting diagonals, so this means that parallelograms, squares, rectangles, and rhombi all have them. 4.

C.

All but general quadrilaterals (i.e. kite) must have two parallel sides. So, trapezoids, parallelograms, squares, rectangles, and rhombi all have at least two. 5.

D.

Only general quadrilaterals and trapezoids can have all differing side lengths. 2.

15) A.

With sides of 4, 6, and 8, the triangle is scalene.

B.

$$4^2 + 6^2 \stackrel{?}{=} 8^2$$

$$52 < 64$$

obtuse

C.

With sides of 16, 16, and 18, the triangle is isosceles.

D.

$$16^2 + 16^2 \stackrel{?}{=} 18^2$$

$$512 > 324$$

acute