Choose NOTA if no other answer is correct.

1. Find the equation of a line perpendicular to the line tangent to the graph of $y = 2x^3 - 3x^2 + 2x - 2$ at $x = 1$.

a. $2x - y = 3$
b. $x - 2y = 3$
c. $2x + y = 1$
d. $x + 2y = -1$
e. NOTA

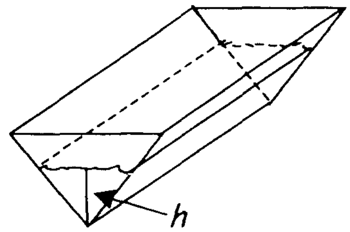
2. Find the point (a,b) on the graph of $f(x) = \sqrt{x-2}$, such that the slope of the tangent line is equal to half of the slope of the tangent line at $x = 3$. What is the value of ab?

a. 12
b. 18
c. 20
d. 24
e. NOTA

3. Function h is defined such that $h(x) = \begin{cases} k\sqrt[3]{x-1} \text{ on } (-\infty,2] \\ mx - 4 \text{ on } (2,\infty) \end{cases}$. If h is differentiable at $x = 2$, find $k + m$.

a. -16
b. -12
c. 8
d. 16
e. NOTA

4. The trough shown has the shape of a regular triangular prism. Each edge of the triangular base measures 4 feet and the length of the trough is 10 feet. Water flows into the trough at 2 cubic feet per minute. At what rate in feet per minute is the height, h, of the water level increasing when the height of the water is 3 feet?



a. $\frac{5\sqrt{3}}{3}$
b. $\frac{\sqrt{3}}{30}$
c. $\frac{1}{20}$
d. $10\sqrt{3}$
e. NOTA

5. The cost in dollars, $C(x)$, to produce 50 notebooks is \$105 and is defined by the formula $C(x) = 0.02x^2 + 0.4x + k$, where x is the number of notebooks produced. Use a linear approximation to estimate the cost of 51 notebooks.

a. \$2.40
b. \$105.24
c. \$107.40
d. \$129
e. NOTA

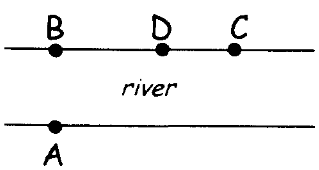
6. Evaluate $\int_{-1}^{2}\left(x^2 + x + \frac{1}{x^2}\right)dx$.

a. -1
b. 3
c. $\frac{20}{3} + \ln(4)$
d. 6
e. NOTA

7. Find $\frac{dy}{dx}$ if $xy+\cos(xy)=x^2$.

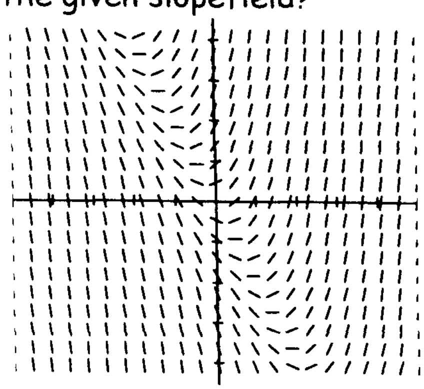
a. $\frac{2x}{1+\sin(xy)}-y$
b. $\frac{2}{1-\sin(xy)}-y$
c. $\frac{2x-y}{x(1+\sin(xy))}$
d. $\frac{2}{1-\sin(xy)}-\frac{y}{x}$
e. NOTA

8. An observer stands at point A on the bank of a river. He needs to get to point C in the least amount of time. He rows a boat at 4 mph to point D and then runs to point C at 6 mph. If point B is directly across the river 2 miles from point A, and C is 8 miles from B, how many miles is point D from point B?



a. $\frac{\sqrt{2}}{4}$
b. $\frac{4\sqrt{5}}{5}$
c. $4\sqrt{2}$
d. $\frac{2\sqrt{5}}{3}$
e. NOTA

9. Which differential equation could be used to create the given slopefield?



a. $y'=x-y$
b. $y'=x\cdot y$
c. $y'=2x+y$
d. $y'=-\frac{x}{y}$
e. NOTA

10. The region bounded by $f(x)=x^2+1, x=0, x=1$, and $y=0$ is rotated about the y-axis. Find the volume of the solid generated.

a. $\frac{3\pi}{2}$
b. $\frac{3\pi}{4}$
c. $\frac{8\pi}{3}$
d. $\frac{5\pi}{4}$
e. NOTA

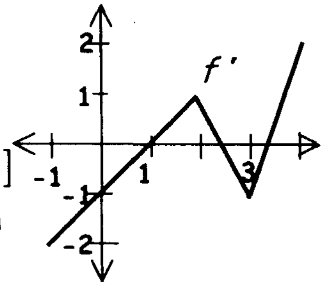
11. Evaluate $\int_0^2 \frac{x}{e^{x^2}}dx$.

a. $\frac{e^2-1}{2e^2}$
b. $\frac{e-1}{2e}$
c. $\frac{1}{2e^4}$
d. $\frac{e^4-1}{2e^4}$
e. NOTA

12. Which expression is equivalent to $\frac{d^2y}{dx^2}$ if $x^3+y^3=8$?

a. $\frac{2xy^3+2x^4}{y^5}$
b. $\frac{-2xy^3+2x^4}{y^5}$
c. $\frac{-16x}{y^5}$
d. $-\frac{x^2}{y^2}$
e. NOTA

Use the given graph of f' to answer questions 13 and 14.



13. Function f is differentiable on $[-1,4]$ and $f(0) = 2$. The graph of f' is made up of line segments with endpoints $(-1,-2)$, $(2,1)$, $(3,-1)$ and $(4,2)$, as shown. How many of the following statements is(are) true?

I. $f(-1) > 2$
II. f has inflection points at $x = 2$ and $x = 3$.
III. f has a local maximum between $x = 2$ and $x = 3$.
IV. $\int_{-1}^{2} f'(x)dx < 0$

a. 1
b. 2
c. 3
d. 4
e. NOTA

14. Find an equation of the line tangent to the graph of f at x = 2. (Use f' above.)

a. $x - y = 2$
b. $x - y = 0$
c. $x + y = -2$
d. $x - y = 1$
e. NOTA

15. Evaluate $\int_{0}^{\pi/3} \tan x \cdot \sec^2 x \, dx$.

a. 3/2
b. $\sqrt{3}/2$
c. 1
d. $\frac{1}{2}$
e. NOTA

16. If $f(x)=\sin(2x)+\cos^2 x$, which of the following is(are) true at $x = \frac{\pi}{6}$?

I. f is decreasing.
II. f is concave down
III. f is continuous

a. I and III only
b. I and II only
c. II and III only
d. I, II, and III
e. NOTA

17. If $f(x) = ax^3 + bx^2 + cx + d, f'(1) = 12$, $f''(1) = 30, f'''(x) = 24$, and $f(0) = -2$, find $a + b + c + d$.

a. -1
b. 1
c. 2
d. 6
e. NOTA

18. What is the value of $\sum_{n=0}^{\infty} \frac{3^{n+1}}{5^n}$?

a. 15/2
b. 3/2
c. 1/2
d. 10/3
e. NOTA

19. Evaluate $\lim_{x \to 0} \frac{\tan x \cdot \sec x - x}{2x^3 + \sin x - x}$.

a. 7/10
b. 3/10
c. 3/11
d. 5/11
e. NOTA

26. The rate of change of a population of bacteria is directly proportional to $\sqrt{y}$, where y is the number of bacteria present at time t. If 1 bacteria is present at $t = 0$ and 25 bacteria are present at $t = 2$, how many bacteria are present at $t = 5$?

a. 110
b. 115
c. 118
d. 121
e. NOTA

27. A particle moves on the x-axis so that its velocity at time t is given by $v(t) = t^2 - 3t$ on $[0,4]$. If $s(t)$ describes its position at time t and $s(0) = 4$, what is the greatest distance between the particle and the origin?

a. 4/3
b. 9/2
c. 17/2
d. 4
e. NOTA

28. The length of a curve from $x = 2$ to $x = 4$ is given by $\int_2^4 \sqrt{4x^2 - 4x + 2}\,dx$.
Which of the following could be an equation for this curve?

a. $y = 4x^2 - 4x + 1$
b. $y = x^2 - x + 5$
c. $y = x^2 + x + 4$
d. $y = 2x - 1$
e. NOTA

29. A curve C is defined by the parametric equations $x = -t^2 - 6t + 2$ and $y = t^3 - 3$. Which of the following are characteristics of the graph of C at the point $(-5,-2)$?

I. The line tangent has a negative slope.
II. The graph of C is concave up.
III. The x-coordinate is decreasing.

a. I, II, and III
b. I and III only
c. III only
d. I and II only
e. NOTA

30. Which of the following series diverge?

I. $\sum_{n=0}^{\infty} \left(\frac{e}{\cos 3}\right)^n$
II. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2 + n}}$
III. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

a. I, II and III
b. I and II only
c. I only
d. II and III only
e. NOTA

ERRATA

ALL ANSWERS Florida Invitational MIDDLETON TIGERS Feb 24, 2007

NO CALCULATORS!

	Algebra I	Geometry	Algebra II	Pre-Calc	Calculus	Statistics	Theta Open	Alpha Open	Statistics Open
	none	1	none	none	1	none	none	none	2
1	C	A	C	B	D	B	B	A	C
2	B	C	B	D	A	C	C	B	D
3	A	C	A	C	A	A	C	A	C
4	D	B	D	A	B	D	D	D	B
5	D	D	C	E	C	B	B	C	A
6	B	D	A	B	~~B~~ E	C	D	C	D
7	B	D	B	D	D	D	A	C	C
8	C	A	B	B	B	C	C	A	~~C~~ → B
9	C	A	C	B	C	C	D	D	A
10	C	C	C	A	A	A	C	B	D
11	A	B	C	C	D	C	B	A	A
12	D	C	D	C	C	B	B	B	B
13	A	A	A	D	D	B	B	B	A
14	B	A	B	C	B	A	A	C	C
15	E	C	C	B	A	D	A	B	D
16	B	D	A	D	C	C	B	A	D
17	C	B	C	A	A	B	B	C	B
18	C	C	C	C	A	D	D	D	A
19	B	B	C	A	D	D	A	D	A
20	B	B	B	D	D	B	D	B	C
21	D	C	D	C	A	B	B	B	D
22	C	D	C	B	C	A	D	C	C
23	C	C	B	A	C	A	A	D	B OR E
24	A	D	C	C	A	B	D	D	D
25	D	B	C	B	D	A	B	C	C
26	B	~~C~~ D	D	A	D	C	C	C	~~A~~ → B
27	B	A	E	C	D	D	D	C	C
28	D	D	D	D	B	D	B	B	C
29	C	C	C	C	A	C	B	A	C
30	C	B	B	D	B	A	D	C	C

D. 1. $y' = 6x^2 - 6x + 2; y'(1) = 2; y(1) = -1; y + 1 = -\frac{1}{2}(x-1); x + 2y = -1$

A. 2. $f'(a) = \frac{1}{2\sqrt{a-2}}; f'(3) = \frac{1}{2}; \frac{1}{2\sqrt{a-2}} = \frac{1}{4}; a = 6; f(6) = 2 = b; ab = 12$

A. 3. $k = 2m - 4; m = -4; \frac{k}{3}(x-1)^{-\frac{2}{3}} = m; k = 3m; 3m = 2m - 4; m = -4; k = -12; m + k = -16$

B. 4. $V = \frac{10\sqrt{3}}{3}h^2; \frac{dV}{dt} = \frac{20\sqrt{3}}{3}h\frac{dh}{dt}; 2 = 20\sqrt{3}\frac{dh}{dt}; \frac{dh}{dt} = \frac{\sqrt{3}}{30}$

C. 5. $C'(x) = 0.04x + 0.4; C'(50) = 2.4; y - 105 = 2.4(51-50); y = 107.4$

B. 6. $\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{x}\right)\Big]_{-1}^{2} = \left(\frac{8}{3} + 2 - \frac{1}{2}\right) - \left(\frac{-1}{3} + \frac{1}{2} + 1\right) = 3$

D. 7. $xy' + y - \sin(xy)(xy' + y) = 2x; (xy' + y)(1 - \sin(xy)) = 2x; y' = \frac{2}{1-\sin(xy)} - \frac{y}{x}$

B. 8. Let x be the distance in miles between B and D and let $f(x)$ be the total time in hours.

$$f(x) = \frac{\sqrt{x^2+4}}{4} + \frac{8-x}{6}; f'(x) = \frac{2x}{8\sqrt{x^2+4}} - \frac{1}{6}; 0 = f'(x); x = \frac{4\sqrt{5}}{5}$$

C. 9. The slopes are positive in Quadrant 1 and negative in Quadrant 3.

A. 10. $V = 2\pi\int_0^1 x(x^2+1)\,dx = 2\pi\left(\frac{x^4}{4} + \frac{x^2}{2}\right)\Big]_0^1 = 2\pi \cdot \frac{3}{4} = \frac{3\pi}{2}$

D. 11. $\int_0^2 x \cdot e^{-x^2}\,dx = \int_0^4 \frac{e^{-u}}{2}\,du = -\frac{e^{-u}}{2}\Big]_0^4 = -\frac{e^{-4}}{2} + \frac{1}{2} = \frac{e^4 - 1}{2e^4}$

C. 12. $3x^2 + 3y^2y' = 0; y' = -\frac{x^2}{y^2}$; $y'' = \frac{-2xy^2 + 2x^2y \cdot -\frac{x^2}{y^2}}{y^4}; = \frac{-2x(y^3 + x^3)}{y^5} = \frac{-16x}{y^5}$

D. 13. All 4 of the statements are true.

B. 14. $f(2) - f(0) = \int_0^2 f'(x)\,dx; f(2) - 2 = 0; f(2) = 2; f'(2) = 1; y - 2 = 1(x-2); x - y = 0$

A. 15. Let $u = \tan(x)$. $\int_0^{\sqrt{3}} u\,du = \frac{u^2}{2}\Big]_0^{\sqrt{3}} = \frac{3}{2}$

C. 16. f is continuous everywhere; $f'(x) = 2\cos(2x) - 2\sin x \cdot \cos x; f'\left(\frac{\pi}{6}\right) > 0$; f is increasing

$$f''(x) = -4\sin(2x) - 4\cos^2 x + 2; f''\left(\frac{\pi}{6}\right) < 0;\ f \text{ is concave down}$$

A. 17. $f'(x) = 3ax^2 + 2bx + c; f''(x) = 6ax + 2b; f'''(x) = 6a; 24 = 6a \rightarrow a = 4; f''(1) = 24 + 2b$
$30 = 24 + 2b \rightarrow b = 3; f'(1) = 12 + 6 + c; 12 = 18 + c \rightarrow c = -6; d = -2; 4 + 3 - 6 - 2 = -1$

A. 18. $S = \frac{a}{1-r}; S = \frac{3}{1-\frac{3}{5}} = 3 \cdot \frac{5}{2} = \frac{15}{2}$

$$\lim_{x\to 0}\frac{\sec x \cdot \tan^2 x + \sec^3 x - 1}{6x^2 + \cos x - 1} =$$

D. 19. $\frac{0}{0} \to L'Hopitals\ Rule:$

$\lim_{x\to 0}\frac{\sec^3 x\cdot 2\tan x+\tan^3 x\cdot \sec x+3\sec^3 x\cdot \tan x}{12x-\sin x}=\lim_{x\to 0}\frac{5\sec^3 x\cdot \tan x+\tan^3 x\cdot \sec x}{12x-\sin x}$ Since the denominator will be 12, we know this is the last step. When differentiating the numerator, any term with tan(x) will become 0, so only terms without tan(x) are needed.

$=\lim_{x\to 0}\frac{5\sec^5 x}{12-\cos x}=\frac{5}{11}$

D. 20. $(x+2)^2=x^2+64-16x\cos\theta \to x-15=-4x\cos\theta \to \cos\theta=\frac{x-15}{-4x}$

$$-\sin\theta\cdot\frac{d\theta}{dt}=-\frac{15}{4x^2}\cdot\frac{dx}{dt};-\sin\theta\cdot\frac{d\theta}{dt}=-\frac{15}{4\cdot 6^2}\cdot 3\to -\frac{\sqrt{55}}{8}\cdot\frac{d\theta}{dt}=-\frac{5}{16}\to\frac{d\theta}{dt}=\frac{\sqrt{55}}{22}$$

A. 21. Let x be the distance from the center of the cone to the base of the cone. Then the radius of the base of the cone is $\sqrt{9-x^2}$ since the radius of the sphere is 3.

$V=\frac{\pi}{3}(9-x^2)(3+x);V'=\frac{\pi}{3}(9-6x-3x^2);V'=0\to x=1;V=\frac{32\pi}{3}$

C. 22. $P=\frac{1}{2}\int_0^{10}2\pi r\cdot\frac{5000}{1+r}dr=5000\pi\int_0^{10}\frac{r}{r+1}dr=5000\pi\int_0^{10}1-\frac{1}{r+1}dr=5000\pi\cdot\left(r-\ln(r+1)\big]_0^{10}\right)=$

$5000\pi(10-\ln 11)\approx 15{,}000\cdot 8=120{,}000$

C. 23. I is not necessarily try because f'' might not change signs. II. True III. True by the Mean Value Theorem for Derivatives.

A. 24. $A(x)=\frac{1}{2}\begin{vmatrix}1 & x & 3\\ 1 & -2 & x+2\\ 1 & x-4 & x\end{vmatrix}=\frac{1}{2}\begin{vmatrix}0 & x+2 & -x+1\\ 1 & -2 & x+2\\ 0 & x-2 & -2\end{vmatrix}=-\frac{1}{2}x^2+\frac{5}{2}x+1;A'(x)=-x+\frac{5}{2};x=\frac{5}{2}$

D. 25. $A=\int_0^{\frac{\pi}{4}}\tan y\,dy=\ln(\sec x)\big]_0^{\frac{\pi}{4}}=\ln\sqrt{2}$

D. 26. $\frac{dy}{\sqrt{y}}=kdt;2y^{\frac{1}{2}}=kt+C;C=2;2\cdot 25^{\frac{1}{2}}=2k+2\to k=4;2y^{\frac{1}{2}}=4t+2;y=121$ at $t=5$.

D. 27. $\int_0^3 v(t)dt=-\frac{9}{2};\int_3^4 v(t)dt=\frac{11}{6}$ The object moves 9/2 to the left and the 11/6 to the right. The greatest distance from the origin is the starting position: 4.

B. 28. $1+\left(\frac{dy}{dx}\right)^2=4x^2-4x+2\to\frac{dy}{dx}=2x-1\to y=x^2-x+C$

A. 29. $\frac{dy}{dx}=\frac{3t^2}{-2t-6}\Big|_{t=1}<0;\frac{d^2y}{dx^2}=\frac{-12t^2-36t+6t^2}{(-2t-6)^2\cdot -2}\Big|_{t=1}>0;\frac{dx}{dt}=-2t-6\big|_{t=1}<0$ All are true.

B. 30. I. Diverges - geometric series with $r>1$.

II. Diverges by the nth term test for divergence.

III. Converges to e^2 using the Maclaurin Series for e^x with x = 2.

CALCULUS Florida Invitational MIDDLETON TIGERS February 24, 2007

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Calculus Question 1

$A=\lim\limits_{x\to -3}\dfrac{x^3+2x^2-5x-6}{x+3}$; $B=\lim\limits_{x\to 7}\dfrac{2x-14}{\sqrt{x+9}-\sqrt{2x+2}}$

$C=\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^x$; $D=\lim\limits_{x\to 0}\dfrac{2\sin x\cdot\cos x-2x^2-2x}{x^2}$

Calculus Question 2

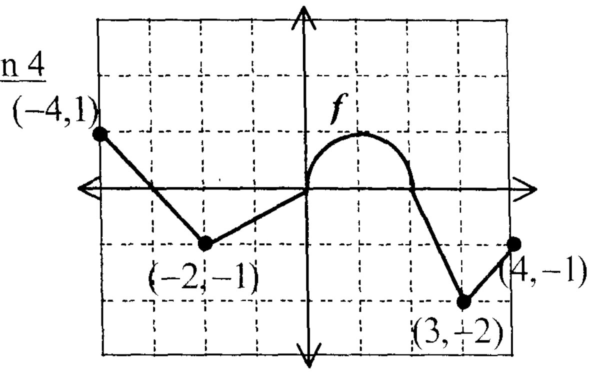
$A=f'\left(\dfrac{\pi}{6}\right)$ if $f(x)=\sin^2(2x)$; $B=g'\left(\sqrt{2}\right)$ if $g(x)=x\cdot\arccos\left(\dfrac{1}{x}\right)$

$C=h'(1)$ if $h(x)=e^{\ln\sqrt{4-x^2}}$; $D=h'(g(2))$ if $h(x)=x^{\ln x}$ and $g(x)=e$

Calculus Question 3

$A=\int_0^{\frac{1}{2}}\dfrac{1}{1-x}dx$; $B=\int_0^{\infty}\dfrac{1}{4+x^2}dx$; $C=\int_0^{1}\dfrac{x}{1+x^2}dx$; $D=\int_0^{1}\dfrac{x^2}{1+x^2}dx$

Calculus Question 4



The graph of f is made up of line segments and a semicircle. Its zeros are $-3, 0,$ and 2. Function f also contains the points labeled on the graph.

$g(x)=\int_0^x f(t)\,dt$ on $[-4,4]$

A = the maximum value of g on $[-4,4]$. B = the minimum value of g on $[-4,4]$.

C = $g(-1)$. D = the sum of the x-coordinates of the inflection points of the graph of g.

Calculus Question 5

Given: $xy^2-x^3y=12$

A = the slope of the tangent line at the point with x-coordinate 1 and positive y-coordinate.

B = the x-coordinate of the point where the tangent line is vertical.

C = the x-coordinate of the point where the tangent line is horizontal.

D = the slope of the tangent line at the point with x-coordinate 1 and negative y-coordinate.

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Calculus Question 6

The point $\left(1,\frac{1}{2}\right)$ is on the graph of $y=f(x)$, and the slope at each point (x,y) on the graph of f is given by $\frac{dy}{dx}=\frac{y^2}{x}$. A = the value of $\frac{d^2y}{dx^2}$ at the point $\left(1,\frac{1}{2}\right)$.

B = k, if $x=k$ is a vertical asymptote of the graph of $y=f(x)$. (positive only)

C = p, if $y=p$ is a horizontal asymptote of the graph of $y=f(x)$. D = $f(e)$.

Calculus Question 7

$A=\frac{d}{dx}(4x\cdot Arcsin\,x)$ at $x=\frac{1}{2}$. $B=\int_0^1 Arctan\,x\,dx$ $C=\lim_{x\to 0}\frac{Arctan\,x}{Arcsin\,x}$

$D=\frac{d}{dx}\left(Arc\sec x^2\right)$ at $x=\sqrt{2}$

Calculus Question 8

Given: $f(x)=\frac{1}{2}x^2, g(x)=\frac{x}{2}$, and $h(x)=-\frac{3}{2}x+2$.

A = the area of the region bounded on the left by $x=0$ and on the right by the graphs of functions g and h.

B = the area of the region bounded above by functions f and h and bounded below by the x-axis.

C = the area between the graphs of functions f and g.

D = the area between the graphs of functions f and h that lies to the left of the y-axis.

Calculus Question 9

$f(x)=\frac{1}{2}x^2$ on $[0,4]$

A = the left Riemann sum approximation of $\int_0^4 f(x)dx$ with 4 equal subintervals.

B = the right Riemann sum approximation of $\int_0^4 f(x)dx$ with 2 equal subintervals.

C = the midpoint Riemann sum approximation of $\int_0^4 f(x)dx$ with 4 equal subintervals.

D = the trapezoidal approximation of $\int_0^4 f(x)dx$ with 4 equal subintervals.

Calculus Question 10

A television camera at ground level is filming the lift-off of a space shuttle that is rising vertically according to the position function $s(t)=\frac{1}{2}t^2$ miles where t is measured in minutes. The camera is $\frac{1}{2}$ mile from the point of lift-off of the shuttle.

A = the rate of change, in radians per minute, of the angle of elevation of the camera 1 minute after lift-off as it follows the path of the shuttle.
B = the velocity, in miles per hour, of the space shuttle 5 minutes after lift-off.
C = the rate of change, in miles per minute, of the distance between the television camera and the shuttle 1 minute after lift-off.
D = the rate of change, in radians per minute, of the angle of elevation of the camera when the velocity of the shuttle reaches 2 miles/min.

Calculus Question 11

Region R_1 is bounded by $y=e^x, x=0, y=0,$ and $x=\ln 2$. Region R_2 is bounded by $y=e^x, x=0,$ and $y=2$.

A = the volume of the solid formed if R_1 is revolved about the line $y=0$.
B = the volume of a solid such that R_1 is the base and all cross sections perpendicular to the x-axis are squares.
C = $a\cdot b$ if the volume of the solid formed when R_2 is revolved about the line $y=0$ is expressed in the form $\pi(\ln a-b)$.
D = $a\cdot b$ if the volume of a solid is $\ln a-b$. The base of the solid is R_2 and all cross sections perpendicular to the x-axis are squares.

Calculus Question 12

The position function, $s(t)=\frac{1}{2}t^3-6t^2+18t-1$, describes the position of an object as it moves on a number line for $0\le t\le 3$.

A = the velocity of the object at $t=1$.
B = the maximum distance from the origin that is reached by the object.
C = the total distance traveled by the object.
D = the acceleration of the object at the instant that it stops.

Calculus Question 13

$$A=\lim_{x\to\frac{\pi}{2}^-}\frac{2\tan x}{1+\sec x};B=\lim_{x\to 0}(1+3x)^{\frac{1}{2x}};C=\lim_{x\to 0}\left(\frac{1}{e^x-1}-\frac{1}{x}\right);D=\lim_{x\to 0^+}\left(e^x-1\right)^x$$

Calculus Question 14

$x(t)=2t^2-5$ and $y(t)=2t+3$ describe the motion of a point moving in the coordinate plane.

A = the velocity of the point at $t=1$. B = the acceleration of the point at $t=1$.

C = the speed of the point at $t=1$. D = $\sin\theta$ where θ is the angle formed by the tangent line and the x-axis at $t=1$.

Calculus Question 15

$\frac{dP}{dt}=0.06P\left(10-\frac{P}{50}\right)$ describes the growth rate of a population, P, of bears in a forest. At t = 0, there were 10 bears in the forest.

A = the maximum number of bears that can be sustained in the forest (the carrying capacity).

B = the number of bears present when the bear population is growing the fastest.

C = $\lim_{t\to\infty}\frac{dP}{dt}$

D = $a\cdot b$ if the value of t when the bear population is growing the fastest is $\frac{a}{b}\ln 7$.

CALCULUS Florida Invitational MIDDLETON TIGERS February 24, 2007

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SOLUTIONS

1. A: $\lim_{x\to -3}(x^2-x-2)=10$

B : $\lim_{x\to 7}\frac{2x-14}{\sqrt{x+9}-\sqrt{2x+2}}\cdot\frac{\sqrt{x+9}+\sqrt{2x+2}}{\sqrt{x+9}+\sqrt{2x+2}}=\lim_{x\to 7}\frac{-2\cancel{(x-7)}\cdot\left(\sqrt{x+9}+\sqrt{2x+2}\right)}{\cancel{-x+7}}=-16$

C : $\ln y=\lim_{x\to\infty}x\cdot\ln\left(1+\frac{2}{x}\right)=\lim_{x\to\infty}\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}=2\to y=e^2$

D: $\lim_{x\to 0}\frac{\sin(2x)-2x^2-2x}{x^2}=\lim_{x\to 0}\frac{-2\cos(2x)-4x-2}{2x}=\lim_{x\to 0}\frac{-4\sin(2x)-4}{2}=-2$

2. A: $f'(x)=2\sin(2x)\cos(2x)\cdot 2\to f'\left(\frac{\pi}{6}\right)=\sqrt{3}$ B: $g'(x)=\frac{1}{\sqrt{x^2-1}}+Arc\cos\frac{1}{x}\to g'(\sqrt{2})=1+\frac{\pi}{4}$

C: $h(x)=\sqrt{4-x^2}\to h'(x)=\frac{-x}{\sqrt{4-x^2}}\to h'(1)=-\frac{\sqrt{3}}{3}$ D: $h'(x)=x^{\ln x}\cdot\frac{2\ln x}{x}\to h'(e)=2$

3. A: $-\ln(1-x)\Big]_0^{\frac{1}{2}}=-\ln\frac{1}{2}-\ln 1=-\ln\frac{1}{2}$ or $\ln 2$ B. $\lim_{b\to\infty}\left(\frac{1}{2}\arctan\frac{x}{2}\Big]_0^b\right)=\frac{\pi}{4}$

C: $\frac{1}{2}\ln(1+x^2)\Big]_0^1=\frac{1}{2}\ln 2$ or $\ln\sqrt{2}$ D: $x-\arctan x\Big]_0^1=1-\frac{\pi}{4}$

4. A: Check -4 and local maxima which occur at -3 and 2. $g(-4)=1; g(-3)=3/2; g(2)=\frac{\pi}{2}$ Max is $\frac{\pi}{2}$.

B: Check 4 and local minimum which occurs at 0. $g(4)=\frac{\pi}{2}-\frac{5}{2}$ and $g(0)=0$ Min is $\frac{\pi}{2}-\frac{5}{2}$.

C: $g(-1)=\int_0^{-1}f(t)dt=-\int_{-1}^{0}f(t)dt=\frac{1}{4}$ D: Inflection points occur at -2, 1, and 3. The sum is 2.

5. A: $x\cdot 2y\frac{dy}{dx}+y^2-x^3\frac{dy}{dx}-y\cdot 3x^2=0\to\frac{dy}{dx}=\frac{3x^2y-y^2}{-x^3+2xy}\to\left.\frac{dy}{dx}\right|_{(1,4)}=-\frac{4}{7}$

B: $-x^3+2xy=0\to -x\left(x^2-2y\right)=0\to y=\frac{x^2}{2}$ Substitute into the original to obtain $x=\sqrt[5]{-48}$.

C: $3x^2y-y^2=0\to y(3x^2-y)=0\to y=3x^2\to x=\sqrt[5]{2}$ D: $\frac{dy}{dx}=\frac{3x^2y-y^2}{-x^3+2xy}\to\left.\frac{dy}{dx}\right|_{(1,-3)}=\frac{18}{7}$

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6. A: $\left.\frac{d^2y}{dx^2}\right|_{(1,1/2)} = \frac{2\cdot 2y\frac{dy}{dx}-y^2}{x^2} = \frac{1\cdot\frac{1}{4}-\frac{1}{4}}{1} = 0$:

B: $\int\frac{dy}{y^2} = \int\frac{1}{x}dx \rightarrow -\frac{1}{y} = \ln(x)+C.\ \ -2 = C \rightarrow y = \frac{-1}{\ln(x)-2}$ and $x = e^2$ is the vertical asymptote. Note: $\lim_{x\to 0^+}\frac{-1}{\ln(x)-2} = 0.$ C: $\lim_{x\to\infty}\frac{-1}{\ln(x)-2} = 0$ so $y = 0$ is a horizontal asymptote. D: $f(e) = \frac{-1}{\ln e - 2} = 1$

7. A: $\left.\frac{4x}{\sqrt{1-x^2}} + 4Arc\sin x\right|_{x=1/2} = \frac{4\sqrt{3}+2\pi}{3}$ C: $\lim_{x\to 0}\frac{\frac{1}{x^2+1}}{\frac{1}{\sqrt{1-x^2}}} = 1$ D: $\left.\frac{1\cdot 2x}{x^2\sqrt{x^4-1}}\right|_{x=\sqrt{2}} = \frac{\sqrt{6}}{3}$

B: $u = \arctan x; dv = dx \rightarrow du = \frac{1}{x^2+1}dx$ and $v = x.$ $x\arctan x\Big]_0^1 - \int_0^1\frac{x}{x^2+1}dx = \frac{\pi}{4} - \frac{1}{2}\ln(x^2+1)\Big]_0^1 = \frac{\pi}{4} - \frac{1}{2}\ln 2$

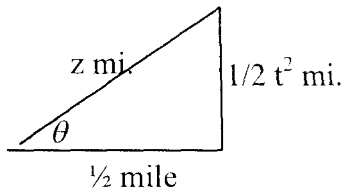
8. A: $A = \frac{1}{2}bh = \frac{1}{2}\cdot 2\cdot 1 = 1$ B: $A = \int_0^1\frac{x^2}{2}dx + \frac{1}{2}\cdot b\cdot h = \frac{1}{6} + \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2} = \frac{1}{4}$

C: $\int_0^1\left(\frac{x}{2} - \frac{x^2}{2}\right)dx = \frac{x^2}{4} - \frac{x^3}{6}\Big]_0^1 = \frac{1}{12}$ D: $\int_{-4}^0\left(\left(-\frac{3}{2}x+2\right) - \frac{x^2}{2}\right)dx = -\frac{3}{4}x^2 + 2x - \frac{x^3}{6}\Big]_{-4}^0 = \frac{28}{3}$

9. A: $f(0)+f(1)+f(2)+f(3) = 0+\frac{1}{2}+2+\frac{9}{2} = 7$ B: $2(f(2)+f(4)) = 2(2+8) = 20$

C: $f(1/2)+f(3/2)+f(5/2)+f(7/2) = 1/8+9/8+25/8+49/8 = 84/8 = 21/2$

D: $1/2\cdot 1\big(f(0)+2f(1)+2f(2)+2f(3)+f(4)\big) = 1/2\cdot 22 = 11$



10. A: $\tan\theta = t^2 \rightarrow \sec^2\theta\frac{d\theta}{dt} = 2t; 2\cdot\frac{d\theta}{dt} = 2\cdot 1 \rightarrow \frac{d\theta}{dt} = 1$

B. $\frac{ds}{dt} = t; \left.\frac{ds}{dt}\right|_{t=5} = 5\frac{\text{mi}}{\text{min}} = 5\cdot 60\text{mph} = 300$ C: $\frac{1}{2}^2 + (\frac{1}{2}t^2)^2 = z^2 \rightarrow t^3 = 2z\frac{dz}{dt}; 1 = 2\cdot\frac{\sqrt{2}}{2}\frac{dz}{dt} \rightarrow \frac{dz}{dt} = \frac{\sqrt{2}}{2}$

D: $v(t) = t \rightarrow t = 2$. $\tan\theta = t^2 \rightarrow \sec^2\theta\frac{d\theta}{dt} = 2t; 17\cdot\frac{d\theta}{dt} = 2\cdot 2 \rightarrow \frac{d\theta}{dt} = \frac{4}{17}$

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11. A: $\pi\int_0^{\ln 2} e^{2x}dx = \pi\cdot\frac{e^{2x}}{2}\Big]_0^{\ln 2} = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$ B: $\int_0^{\ln 2} e^{2x}dx = \frac{e^{2x}}{2}\Big]_0^{\ln 2} = 2 - \frac{1}{2} = \frac{3}{2}$

C: $\pi\int_0^{\ln 2}\left(4 - e^{2x}\right)dx = \pi\cdot\left(4x - \frac{e^{2x}}{2}\right)\Big]_0^{\ln 2} = \pi\cdot\left(4\ln 2 - 2 + \frac{1}{2}\right) = \pi\cdot\left(\ln 16 - \frac{3}{2}\right); 16\cdot\frac{3}{2} = 24$

D: $\int_0^{\ln 2}\left(2 - e^{x}\right)^2 dx = \int_0^{\ln 2}\left(4 - 4e^{x} + e^{2x}\right)dx = 4x - 4e^{x} + \frac{e^{2x}}{2}\Big]_0^{\ln 2} = \ln 16 - 8 + 2 - (0 - 4 + \frac{1}{2}) = \ln 16 - \frac{5}{2}; 16\cdot\frac{5}{2} = 40$

12. A: $v(t) = \frac{3}{2}t^2 - 12t + 18 \rightarrow v(1) = \frac{15}{2}$

B: $v(t) = 0 \rightarrow t = 2; s(0) = -1; s(2) = 15; s(3) = \frac{25}{2} \rightarrow 15$ is the max

C: See part B. $16 + \frac{5}{2} = \frac{37}{2}$ D: $a(t) = 3t - 12 \rightarrow a(2) = -6$

13. A: $\lim_{x\to\frac{\pi}{2}^-} = \frac{2\sec^2 x}{\sec x\cdot\tan x} = \lim_{x\to\frac{\pi}{2}^-}\frac{2}{\sin x} = 2$ B: $\ln y = \lim_{x\to 0}\frac{\ln(1+3x)}{2x} = \frac{3}{2} \rightarrow y = e^{\frac{3}{2}}$

C: $\lim_{x\to 0}\left(\frac{x - e^{x} + 1}{xe^{x} - x}\right) = \lim_{x\to 0}\left(\frac{1 - e^{x}}{xe^{x} + e^{x} - 1}\right) = \lim_{x\to 0}\left(\frac{-e^{x}}{xe^{x} + e^{x} + e^{x}}\right) = -\frac{1}{2}$

D: $\ln y = \lim_{x\to 0^+}\frac{\ln(e^{x} - 1)}{\frac{1}{x}} = \lim_{x\to 0^+}\frac{e^{x}\cdot -x^2}{e^{x} - 1} = \lim_{x\to 0^+}\frac{e^{x}\cdot -2x + e^{x}\cdot -x^2}{e^{x}} = 0 \rightarrow y = 1$

14. A: $\left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=1} = \left.\frac{1}{2t}\right|_{t=1} = \frac{1}{2}$ B: $\frac{\frac{d}{dt}\left(\frac{1}{2t}\right)}{4t} = \frac{\frac{-1}{2t^2}}{4t} = \frac{-1}{8t^3} = -\frac{1}{8}$ at $t = 1$.

C: $\left.\sqrt{(4t)^2 + 4}\right|_{t=1} = \sqrt{20} = 2\sqrt{5}$ D: $\frac{dy}{dx} = \frac{1}{2} = \tan\theta \rightarrow \sin\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

15. A: $\frac{dP}{dt} = 0.6y\left(1 - \frac{y}{500}\right) \rightarrow P(t) = \frac{500}{1 + 49e^{-0.6t}} \rightarrow \lim_{t\to\infty} P(t) = 500$

B: The population grows the fastest at 1/2 the carrying capacity or 250. C: In logistics growth, the growth rate approaches 0 as time increases. 0

D: $250 = \frac{500}{1 + 49e^{-6t}} \rightarrow 1 = \frac{2}{1 + 49e^{-0.6t}} \rightarrow 1 + 49e^{-0.6t} = 2 \rightarrow e^{-0.6t} = \frac{1}{49} \rightarrow t = \frac{\ln 49}{\frac{3}{5}} = \frac{10\ln 7}{3}; a\cdot b = 30$