Properties for Multiplication

The CLOSURE property for Multiplication	For all real numbers a and b: ab is a unique real number
	$\forall a,b \in \Re, \exists! c \in \Re; (ab = c)$
The ASSOCIATIVE property for Multiplication	For all real numbers a, b, and c: $(ab)c = a(bc)$
	$\forall a,b,c \in \Re; (ab)c = a(bc)$
The COMMUTATIVE property for Multiplication	For all real numbers a and b: $ab = ba$
	$\forall a,b \in \Re; \quad ab = ba$
IDENTITY Property of Multiplication	There is a unique real number 1 such that for every real number a, $a \cdot 1 = a$ and $1 \cdot a = a$.
	$\forall a \in \Re; \quad (a \cdot 1 = a) \wedge (1 \cdot a = a)$
Property of RECIPROCALS	For every non-zero real number a, there is a unique real number $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$
	$\forall a \in \Re \wedge a \neq 0, \exists! \frac{1}{a}; \quad (a \cdot \frac{1}{a} = 1) \wedge \left(\frac{1}{a} \cdot a = 0\right)$

Definition of Division

Definition of Division	For all real numbers a and b, the **quotient** $a \div b$ is defined by $a \div b = a \cdot \frac{1}{b}$ for $b \neq 0$ to multiply by *b*, multiply by the reciprocal of b.
	$\forall a,b \in \Re, b \neq 0; \quad a \div b = a \cdot \frac{1}{b}$

Rule for Multiplication by -1

Theorem: Multiplication by -1	For all real numbers a: $-1 \cdot a = -a$
Theorem: Negative times a positive	$\forall a \in \Re; \quad -1 \cdot a = -a$
Theorem: Negative times a negative is a positive	$-1 \cdot -1 = -(-1) = 1$

Rules for Multiplication and Addition

The DISTRIBUTIVE property (of multiplication with respect to addition)	For all real numbers a, b, and c: $a(b+c) = ab + ac$
	$\forall a,b,c \in \Re; \quad a(b+c) = ab + ac$

Sample Quiz on Axioms for Multiplication

1. Find a solution set for $4x = 8$

$4x = 8$	Given
$\frac{1}{4} \cdot (4x) = \frac{1}{4} \cdot 8$	
$\left(\frac{1}{4} \cdot 4\right) x = \frac{1}{4} \cdot 8$	
$1 \cdot x = \frac{1}{4} \cdot 8$	
$x = \frac{1}{4} \cdot 8$	
$x = 2$	

2. Find a solution set for $x \div 3 = 9$

$x \div 3 = 9$	Given
$x \cdot \frac{1}{3} = 9$	
$\left(x \cdot \frac{1}{3}\right) \cdot 3 = 9 \cdot 3$	
$x \cdot \left(\frac{1}{3} \cdot 3\right) = 9 \cdot 3$	
$x \cdot 1 = 9 \cdot 3$	
$x = 9 \cdot 3$	
$x = 27$	

3. Find a solution set for $6 - x = 1$ (Additive Solution)

$6 - x = 1$	Given
$6 + -x = 1$	
$(6 + -x) + x = 1 + x$	
$6 + (-x + x) = 1 + x$	
$6 + 0 = 1 + x$	
$6 = 1 + x$	
$1 + x = 6$	
$x + 1 = 6$	
$(x + 1) + -1 = 6 + -1$	
$x + (1 + -1) = 6 + -1$	
$x + 0 = 6 + -1$	
$x = 6 + -1$	
$x = 5$	

4. Find a solution set for $6 - x = 1$ (Multiplicative Solution)

$6 - x = 1$	Given
$6 + -x = 1$	
$6 + (-1 \cdot x) = 1$	
$-6 + (6 + (-1 \cdot x)) = -6 + 1$	
$(-6 + 6) + -1 \cdot x = -6 + 1$	
$0 + -1 \cdot x = -6 + 1$	
$-1 \cdot x = -6 + 1$	
$-1 \cdot x = -5$	
$-1 \cdot (-1 \cdot x) = -1 \cdot -5$	
$(-1 \cdot -1) \cdot x = -1 \cdot -5$	
$1 \cdot x = 5$	
$x = 5$	