Relation
A relation on the elements of a set, S, is a set of ordered pairs of those elements.

Function
A function on the elements of a set, S, is a set of ordered pairs of elements with a unique first coordinate.

General Properties of Relations: *R*

R is Reflexive on S	$\forall a \in S, aRa$
R is Symmetric on S	$\forall a, b \in \mathrm{S}; aRb \Rightarrow bRa$
R is Transitive on S	$\forall a, b, c \in S; aRb \wedge bRc \Rightarrow aRc$
Irreflexive	$\forall a \in S; \neg(aRa)$
Antisymmetric	$\forall a \in S; aRb \wedge bRa \Rightarrow a = b$

Examples for reflexive
The relation $\leq$ is reflexive on $\Re$. $r \leq r$ is true for every real number.
The relation = is reflexive on $\Re$. $r = r$ is true for every real number.
The relation $\subseteq$ is reflexive on the **power set** of any set.

Non-examples
The relation "$<$" is not reflexive on $\Re$
The relation "is a sister of" is not reflexive on the set of all women.

Examples for symmetric
The empty relation is symmetric
The equals relation is symmetric (i.e. a=b → b=a)

Not examples
The "is a sister" relationship is **symmetric** for the set of all women **BUT**
the relation is **not** symmetric for the set of all people.

Examples for antisymmetric
The empty relation is antisymmetric
The $<$ relation is antisymmetric. (Do you know the definition of $a \to b$?)
The $\leq$ relation on $\Re$ is antisymmetric. $a \leq b \wedge b \leq a \Rightarrow a = b$

Warning: antisymmetric is NOT the negation of symmetry. "=" is both antisymmetric and symmetric.

Examples of Transitive
$<, \leq, >, \geq, =$ (generally all ordering relations)

Types of Relations

Name	Properties	Examples
Order Relation	antisymmetric, transitive	
Equivalence Relation	reflexive, symmetric, transitive	equality, congruence, similarity

Definition

a < b	$\forall a, b \in \Re; a < b \Leftrightarrow b - a \in \Re^+$
$a \leq b$	$a \leq b \Leftrightarrow a < b \vee a = b$

Equivalence Relation

An Equivalence Relation, *R*, is a relation with the following properties: Reflexive, Symmetric and Transitive.

Reflexive	For each element in the set $a \in S, aRa$
	$\forall a \in S; aRa$
Symmetric	For all elements *a* and *b* in set, *S*; if aRb then bRa
	$\forall a, b \in S; aRb \Rightarrow bRa$
Transitive	For elements a, b, c which are elements of set, *S;* if aRb and bRc then aRc
	$\forall a, b, c \in S; (a\,R\,b) \wedge (b\,R\,c) \Rightarrow a\,R\,c$

The relation **equality** (=) and **congruence** $(\cong)$ are equivalence relations. Equality is generally used for numbers. Congruence is generally used for geometric shapes The relation **similarity** (~) is not an equivalence relation.

Properties for Equality

Reflexive	For each element in the set $a \in S, a = a$
	$\forall a \in S; a = a$
Symmetric	For all elements *a* and *b* in set, *S*; if $a = b$ then $b = a$
	$\forall a, b \in S; a = b \Rightarrow b = a$
Transitive	For elements a, b, c which are elements of set, *S;* if $a = b$ and $b = c$ then $a = c$
	$\forall a, b, c \in S; (a = b) \wedge (b = c) \Rightarrow a = c$
The ADDITION property for Equality	For all elements, a,b,c, in set, *S*, if a = b then a + c = b + c.
	$\forall a, b, c \in S; a = b \Rightarrow a + c = b + c$
The MULTIPLICATION property for Equality	For all elements, a,b,c, in set, *S*, if a = b then $ac = bc$
	$\forall a, b, c \in S; a = b \Rightarrow ac = bc$