Probability Review

1. What is the sample space when two fair dice are rolled?
2. Roll a die twice. Observe the numbers landed on.
    a) What is the probability of observing two 5's?
    b) What is the probability of observing at least one 5?
    c) What is the probability of observing exactly one 5?
3. Let P(A) = 0.5, P(B) = 0.7, P(A and B) = 0.4, find the probability that
    a) Either A or B will occur
    b) Neither A nor B will occur
    c) A will occur, and B does not occur
    d) A will occur, given that B has occurred
    e) A will occur, given that B has not occurred
4. A coin is flipped three times. Event A consists of observing exactly 2 heads. Event B consists of observing one or more tails.
    a) List the members of event A.
    b) List the members of event B.
    c) Find the union of event A and event B.
    d) Find the intersection of event A and event B.
    e) Find P(A | B)
5. Let $P(A \cap B) = \frac{3}{6}$ $P(A^c \cap B) = \frac{1}{6}$ $P(A \cap B^c) = \frac{1}{6}$
    a) Draw and label a Venn Diagram for events A and B.
    b) $P(A^c \cap B^c) =$
    c) $P(A^c \cup B^c) =$
    d) Are A and B mutually exclusive (disjoint)?
    e) Are A and B independent?
6. A player rolls two dice. The players receive $10.00 for each dot on the face of the two dice. For example, he will be paid $9.00 if the dice show a 4 and a 5. Let *X* = the number of dollars won each game.
    a) Is *X* a discrete or continuous random variable?
    b) Show the probability distribution of *X*.
    c) Find the expected value of *X*.
    d) Find the variance of the pay off.

7. Roll two dice and observe the sum. Define the events as follows:

   A = {the value is even}

   B = {the value is odd}

   C = {the value is less than 6}

   D = {the value is greater than or equal to 6}

   a) Write the sample space for the observed sum.

   b) Write the probability distribution for the observed sum.

   c) Are events A and C mutually exclusive (disjoint)?

   d) Are events A and C independent?

   e) Find $A \cup C$

   f) Calculate $P(A \cup C)$

   g) Find $B \cup D$

   h) Calculate $P(B \cup D)$

8. Let *X* = the actual weight of a 2-lb. bag of rice. The mean amount of rice is 32 ounces, with standard deviation is 0.25 ounces.

   a) Is *X* a discrete or continuous variable?

   b) What percentage of the population will be within 31.5 and 32.5 ounces?

9. A researcher surveyed students majoring in business and asked what type of car they own.

	Bought New	Bought Used
Male	21	32
Female	44	15

   a) What is the probability that a female student purchased a new car?

   b) What is the probability that a male student purchased a used car?

   c) What is the probability that a student purchased a new car?

   d) What is the probability that a student is female?

   e) Given that a student is female what is the probability that she purchased a used car?

   f) Given that a car is new what is the probability that the owner is a male?

10. A new detergent is found to remove excess dirt and stains satisfactorily on 88% of the items washed. Assume that 3 items are to be washed with the new detergent.

    a) What is the probability of satisfactory results on all 3 items?

    b) What is the probability that exactly two items are cleaned satisfactorily?

    c) What is the probability that none are cleaned satisfactorily?

11. Given three outcomes E_1, E_2, and E_3 with probabilities $P(E_1) = 0.05$, $P(E_2) = 0.55$, and $P(E_3) = 0.4$ Let event $A=\{E_1,E_3\}$, and $B=\{E_2, E_3\}$
    a) Find the probability of A union B.
    b) Find the probability of A intersect B.
    c) Find the probability of A given B.
    d) Find the probability of B given A.
    e) Are A and B independent?

12. For an interstellar space truck stop the probability that the owner comes from the planet Orkbak is 0.62. Of the Orkbakians, 88% are giants. What is the probability that the owner of a randomly chosen interstellar truckstop is an Orkbakian giant?

13. Three friends are trying to decide who gets the last doughnut. They decide on the following scheme: each will flip a fair coin and whoever gets the unique result will win the doughnut (if the result is HTT then the first wins; if the result is HTH then the second wins). If all come out the same, they will feed the doughnut to the birds.
    a) What are the probabilities of each one winning?
    b) What is the probability that the birds get it?

14. A study found that on a given day 72% of women and 46% of men made their beds. In the U.S. women comprise 52% of adults. Find the following:
    a) The percentage of all adults who are men and made their beds.
    b) The percentage of all adults who are women and made their beds.
    c) The percentage of all adults who made their beds.
    d) If a person has made their bed, what is the probability that that person is a woman?

15. You can insure a $25,000 diamond for its total value by paying a premium of *C* dollars. If the probability of theft in a given year is 0.20, what premium should the insurance company charge if it wants the expected gain to be equal to $1000?

16. Microcomputers are shipped to the University bookstore from three factories A, B, and C. You know that factory A produces 20% defective microcomputers, whereas B produces 10% defectives and C only 5% defectives. The manager in the store receives a new shipment of microcomputers and discovers that 40% are from factory C, 40% are from factory B, and 20% are from factory A. (Hint: make a tree diagram)
    a) What is the probabilities of finding a defective microcomputer in this shipment?
    b) Are the events "microcomputer comes from factory A" and "microcomputer comes from factory B" mutually exclusive? Are they independent?
    c) Suppose the manager randomly selects one microcomputer, and discovers that it is defective. What is the probability that it came from factory A?

17. Let *X* be a discrete random variable with probabiltiy distribution:

X	*p(x)*
1	.15
2	.40
3	.10
4	.35

a) Find $P(X = 2)$.

b) Find $P(X \leq 2)$.

c) Find $P(X < 2)$

d) Find $P(2 \leq X \leq 4)$

18. Let *X* be a discrete random variable with probabiltiy distribution:

X	*p(x)*
1	.10
2	.45
3	.15
4	.25

a) Does this assignment define a proper probability distribution? Explain.

19. Investing $100 in a project will yield a net return of $16, $20 or $26 with respective probabilities 0.3, 0.5 and 0.2. Let *X* be the random variable that represents the net return.

a) Write down the probability distribution of the variable *X.*

b) Find the expected return for the project, and the standard deviation.

c) Define $Y = 3X + 2$. Specify the probability distribution of *Y*.

Probability Review

1. What is the sample space when four fair coins are tossed?

2. Flip a coin four times. Observe whether it lands on heads or tails.
   a) What is the probability of observing four heads?
   b) What is the probability of observing at least one head?
   c) What is the probability of observing exactly one head?

3. For a family living in Southern Mississippi, the probability of owning a dog is 0.4, the probability of owning a cat is 0.5, and the probability of owning both a cat and a dog is 0.12.
   a) Use a Venn Diagram to show the Union of the two events.
   b) Use a Venn Diagram to show the Intersection of the two events.
   c) What is the probability that a family living in Southern Mississippi does not own a dog?
   d) What is the probability that a family living in Southern Mississippi owns a cat or a dog?
   e) What is the probability that a family living in Southern Mississippi owns neither a cat nor a dog?
   f) Are events A and B independent?
   g) Are events A and B mutually exclusive (disjoint)?

4. Roll a die. Let S = {1, 2, 3, 4, 5, 6 }
   Let event A={ 1, 2, 3, 4 } and event B= { 2, 3, 4, 5, 6 }. Find the following:
   a) Members of the set $A \cap B$
   b) $P(A \cap B)$
   c) Members of the set $A \cap B^c$
   d) $P(A \cap B^c)$
   e) Members of the set $A^c \cap B$
   f) $P(A^c \cap B)$
   g) Members of the set $A^c \cap B^c$
   h) $P(A^c \cap B^c)$

5. Draw and label each of the following on a Venn Diagram:
   a) $(A \cap B)$
   b) $(A^c \cap B)$
   c) $(A \cap B^c)$

6. A player rolls two dice. The players receive \$1.00 for each dot on the face of the two dice. For example, he will be paid \$9.00 if the dice show a 4 and a 5. Let *X* = the number of dollars won each game.
    a) Is *X* a discrete or continuous random variable?
    b) Show the probability distribution of *X*.
    c) Find the expected value of *X*.
    d) Find the variance of the pay off.

7. Roll two dice and observe the sum. Define the events as follows:
    A = {the value is even}
    B = {the value is odd}
    C = {the value is less than 6}
    D = {the value is greater than or equal to 6}
    a) Write the sample space for the observed sum.
    b) Write the probability distribution for the observed sum.
    c) Are events A and B mutually exclusive (disjoint)?
    d) Are events A and B independent?
    e) Find $A \cup D$
    f) Calculate $P(A \cup D)$
    g) Find $B \cup C$
    h) Calculate $P(B \cup C)$

8. A packaging company distributes quarter-pound beef patties to local fast-food restaurants. The restaurants demand the weights to be 4.000 ± 0.02 ounces. If the patties are too small, the customers will complain, and if they are too big, there will be too few of them in a box and the business will lose money. If the standard deviation of the patties is 0.009, what percentage of the patties will not meet the expectation?

9. The probability that the space shuttle is launched on the designated day is 80%. Assume that shuttle launches are independent from each other. Suppose four launches are scheduled in the next three months.
    a) What is the probability that each one is launched on the designated day?
    b) What is the probability that exactly one is launched on the designated day?
    c) What is the probability that the first two are launched on the designated day, but the last two are not?

10. A researcher surveyed students majoring in business and asked what type of car they own.

	Bought New	Bought Used
Male	15	32
Female	44	12

    a) What is the probability that a female student purchased a new car?
    b) What is the probability that a male student purchased a used car?
    c) What is the probability that a student purchased a new car?
    d) What is the probability that a student is female?
    e) Given that a student is female what is the probability that she purchased a used car?
    f) Given that a car is new what is the probability that the owner is a male?

11. It is estimated that 22% of the households in America are headed by a single adult. Out of these single adult households, 21% are headed by men. What percent of American households are headed by single men?

12. A craftsperson makes stuffed animals for sale. Eighty-five percent of the animals she makes are bears while the remaining 15% are rabbits. Suppose she sells 75% of her bears at craft shows but only 50% of her rabbits are sold at craft shows. What proportion of her animals does she sell at craft shows? (Hint: use a tree diagram)

13. Three friends are trying to decide who gets the last doughnut. They decide on the following scheme: each will flip a fair coin and whoever gets the unique result will win the doughnut (if the result is HTT then the first wins; if the result is HTH then the second wins). If all come out the same, they will feed the doughnut to the birds.
    a) What are the probabilities of each one winning?
    b) What is the probability that the birds get it?

14. A study found that on a given day 76% of women and 46% of men made their beds. In the U.S. women comprise 51% of adults. Find the following:
    a) The percentage of all adults who are men and made their beds.
    b) The percentage of all adults who are women and made their beds.
    c) The percentage of all adults who made their beds.
    d) If a person has made their bed, what is the probability that that person is a woman?

15. You can insure a $25,000 diamond for its total value by paying a premium of *C* dollars. If the probability of theft in a given year is 0.10, what premium should the insurance company charge if it wants the expected gain to be equal to $1000?

16. Microcomputers are shipped to the University bookstore from three factories A, B, and C. You know that factory A produces 20% defective microcomputers, whereas B produces 10% defectives and C only 5% defectives. The manager in the store receives a new shipment of microcomputers and discovers that 40% are from factory C, 40% are from factory B, and 20% are from factory A. (Hint: make a tree diagram)
    a) What is the probabilities of finding a defective microcomputer in this shipment?
    b) Are the events "microcomputer comes from factory A" and "microcomputer comes from factory B" mutually exclusive? Are they independent?
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17. Let *X* be a discrete random variable with probabiltiy distribution:

X	*p(x)*
1	.20
2	.40
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    a) Find $P(X = 2)$.
    b) Find $P(X \le 2)$.
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18. Let *X* be a discrete random variable with probabiltiy distribution:

X	*p(x)*
1	.20
2	.40
3	.20
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    a) Does this assignment define a proper probability distribution? Explain.

19. Investing $100 in a project will yield a net return of $8, $10 or $12 with respective probabilities 0.2, 0.6 and 0.2. Let *X* be the random variable that represents the net return.
    a) Write down the probability distribution of the variable *X*.
    b) Find the expected return for the project, and the standard deviation.
    c) Define $Y = 2X + 1$. Specify the probability distribution of *Y*.

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