PROBLEMS ON REAL NUMBERS

I.5 For a positive integer n the symbol n! means the product of the integers from 1 to n. Then $n!/(n-2)!$ =

(a) $n+2$ (b) $n^2 - n$ (c) $n^2 - 2n$ (d) $n/(n-2)$ (e) $2n - 1$

I.12 Which one of these is a rational number?

(a) $2^{1/2}$ (b) e (c) π (d) $e^{\pi i}$ (e) $2^{1/2} + 3^{1/2}$

I.15 Define the operation (*) by $a*b = ab - a - b$. Then $(3*2)*3$ = (a) 0 (b) -1 (c) 18 (d) 2 (e) none of these

I.21 When the decimal .321212121... is written in lowest terms as a fraction a/b, a > 0, then a + b =

(a) 1321 (b) 1309 (c) 131 (d) 321 (e) 218

I.33 $(5 + 2(6)^{1/2})^{1/2} - (5 - 2(6)^{1/2})^{1/2}$ equals

(a) $2^{1/2}$ (b) $6^{1/2}$ (c) $2(6)^{1/2}$ (d) 3 (e) $2(2)^{1/2}$

I.39 A function f is additive if $f(x + y) = f(x) + f(y)$ for all x and y. Consider the four functions:

$f_1(x) = 3x$; $f_2(x) = 2x + 1$; $f_3(x) = x^2$; $f_4(x) = x^{1/2}$

The sum of the numbers k for which $f_k(x)$ is additive is

(a) 0 (b) 1 (c) 3 (d) 8 (e) 10

II.9 If the number 314_5 to the base 5 is expressed to the base 7 then the sum of its digits is

(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

III.9 Let N be a 4 digit number abcd and M the 4 digit number dcba obtained by reversing the digits of N (decimal base assumed). Then M - N is an even number only if

(a) d - a is even (b) c - b is odd (c) ad = bc (d) a + d = b + c (e) a + b + c + d = 0

III.24 Let $f(x) = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$; then $f(1/3)$ = (a) $5^3/3^5$ (b) $(4/3)^5$ (c) $1 + (2/3)^5$ (d) $3 - (1/3)^5$ (e) $(5/3)^4$

III.32 The minimum value of the quotient of a (base ten) number of 3 non-zero digits divided by the sum of the digits is

(a) 10 (b) 19/2 (c) 199/19 (d) 21/2 (e) 119/11

III.33 When the number 2^{1000} is divided by 13, the remainder in the division is (a) 1 (b) 2 (c) 3 (d) 7 (e) 11

IV.30 The units digit of 227 is 7; of 356 is 6. What is the units digit of the number 3^{1000} ?

(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

VII.11 Given that B and C are integers from 0 to 9 and the product of the three digit integer B2C and the two digit integer CB is 23,871 then B + C =

(a) 10 (b) 9 (c) 5 (d) 13 (e) 7

VIII.5 Given the mathematical operations * and # defined by $a*b = a + 2b$ and $a\#b = a + b - 2$, which of the properties A,B,C,D are true?

A: * is commutative B: # is commutative
C: * is associative D: # is associative

(a) A,B only (b) B,C only (c) B,D only (d) all (e) none

VIII.24 Which product is closest to 1?

(a) 1.01X.9999999 (b) 1.001X0.999999 (c) 1.0001x0.99999
(d) 1.00001x0.9999 (e) 1.000001x0.999