PROBLEMS ON THE ROOTS OF POLYNOMIAL EQUATIONS

I.9 Suppose f is a polynomial of degree 5, $f(0) = 3$, $f(1) = 1$, $f(2) = -1$, $f(3) = -1/2$. What can we say about the roots of f? (a) there may not be any (b) there is a negative root (c) there is a root greater than 3 (d) there is a root between 1 and 2 (e) there is a root between 2 and 3

I.14 Which of the following polynomials has the property that if r is a root (real or complex) then so is -r?

(a) $x^5 + 3x^2 + 6$ (b) $x^7 - x^5 + 2x^3 - x$ (c) $x^4 + 2x^2 + x - 1$ (d) $x^4 + x^2 + x$ (e) none of (a),(b),(c),(d)

I.24 Which of the following polynomials has the property that the sum of its roots (real and complex) is 6 and the product of its roots is 3?

(a) $x^3 - 6x^2 + 3x - 1$ (b) $x^3 - 3x^2 + 6x - 1$ (c) $x^3 - 6x^2 + 2x + 3$ (d) $x^3 - 6x^2 + 2x - 3$ (e) $x^2 - 11$

I.36 If r_1, r_2, ... are the values of x for which $x^8 + 4x^6 - \pi x^4 + 3x^2 - 2^{1/2}$ has a minimum, then $r_1 + r_2 + \ldots +$ is

(a) 0 (b) 1 (c) -1 (d) 2 (e) an irrational number.

II.15 If p is a polynomial with real coefficients and no multiple roots, which of the following could be true?

(a) p is of degree 6 and has exactly 3 real roots.
(b) p is of degree 7 and has no real roots.
(c) p is of degree 9 and has exactly 1 real root.
(d) p is of degree 17 and has exactly 2 real roots.
(e) p is of degree 6 and has exactly 7 real roots.

II.26 If $p(x) = x^{2n} + ax^{2n-1} + \ldots$ is a polynomial of degree 2n which has as roots the numbers $+1,-1,+2,-2,+3,-3,\ldots,+n,-n$ then what is a? (a) $n(n + 1)/2$ (b) 0 (c) 1 (d) n (e) -1

II.28 Suppose p is a polynomial of degree 6 with real coefficients such that $p(-1) = 3$, $p(0) = -2$, $p(4) = 1$, $p(6) = -2$. Let n be the number (counting multiplicities) of real roots of p. Which of the following is the most we can say about n? (a) $n \leq 6$ (b) $n \geq 1$ (c) $3 \leq n \leq 6$ (d) $4 \leq n \leq 6$ (e) $2 \leq n \leq 4$

II.34 Suppose a and b are integers such that the polynomial $x^6 + ax^5 + bx^4 + ax^3 - bx^2 - 1$ has a positive integral root. What is a? (a) 1 (b) -1 (c) 0 (d) 2 (e) there are several values depending on b

III.34 The roots of $64x^3 - 144x^2 + 92x - 15 = 0$ are in arithmetic progression. The difference between the largest and smallest roots is

(a) 2 (b) 1 (c) 4 (d) 3/8 (e) 1/4.

IV.24 Given the cubic equation $x^3 + Ax^2 + Bx + C = 0$, if S is the sum of the roots and P is the product of the roots then (a) $C = PS$ (b) $B = P + S$ (c) $A = P/S$ (d) $AB = PS$ (e) $A + C = -P - S$

IV.28 If $x^{70} + x^{10} + x - 5$ is divided by $x^2 - 1$ the remainder is (a) $x - 3$ (b) $2x + 7$ (c) $5x - 1$ (d) $x + 2$ (e) $5 - 2x$.

V.3 If X = 1 and X = -1 are solutions of F(X) = 0 where F(X) is a polynomial, then a divisor of F(X) is

(a) X (b) $X^2 + 1$ (c) 2X + 1 (d) X + 2 (e) $X^2 - 1$

VI.26 The equation $x^2 + x + 1 = A$ has exactly one real root provided A = (a) 0 (b) 1 (c) -1 (d) 2/3 (e) 3/4

VII.23 If $i^2 = -1$ then among all complex numbers z = a + bi such that $z^3 = i$, the largest value of b is

(a) 1/3 (b) 1/4 (c) 2/3 (d) 1/2 (e) 3/2.

VII.25 If the polynomial $x^3 + bx^2 + cx + d$ has all real roots, the sum of the roots is 4, the product of the roots is -6, and one of the roots is 3, then c =

(a) -3 (b) 5 (c) -4 (d) 1 (e) cannot tell from the given information.

VIII.15 Given that one root of the equation $2x^4 + 5x^3 - 10x^2 + 5x - 12 = 0$ is x = i (where $i^2 = -1$) then the sum of all real roots is (a) 0 (b) 5 (c) -5 (d) -5/2 (e) 13/2

IX.2 If x = i (where $i^2 = -1$) is a root of a polynomial p(x) = 0 with real number coefficients then p(x) is divisible by (a) $x^2 - 2x + 2$ (b) $x^2 + x + 1$ (c) $x^2 - 2x - 4$ (d) $x^2 - x + 1$ (e) $x^2 + 1$

X.4 The sum of the real roots of the equation $x^5 - x^4 + 2x^3 - 2x^2 + x - 1 = 0$ is (a) 0 (b) 1 (c) 2 (d) 5 (e) 9