PROBLEMS ON INEQUALTIES

I.27 If a and b are non-negative numbers, when is

$$a^3b + ab^3 \geq a^4 + b^4 \text{ ?}$$

(a) only when $a = 0$ (b) for all a and b (c) only for $a \geq b$ (d) only when $a = b$ (e) only for $a + b = 1$

I.41 If a and b are positive numbers let $A = (a + b)/2$, $B = 2ab/(a + b)$, $C = (ab)^{1/2}$.

(a) $A \geq C \geq B$ always (b) $A \geq B \geq C$ always
(c) $C \geq A \geq B$ always (d) $A > B$ always
(e) The order of A,B,C depends on a,b.

II.2 Given $f(x) = 6x^3 - 5x^2 + x$, in which case is $f(x) > 0$ for both values of x?

(a) 99/301 and 101/200 (b) 99/301 and 99/199
(c) 1/100 and 99/199 (d) 151/300 and 149/300
(e) -1/100 and 301/99

V.21 If the two digit numbers '2X' and 'Y1' are added, then the result is greater than 100 provided

(a) $X + Y \geq 15$ (b) $10Y + X \geq 80$ (c) $10X + Y \geq 120$
(d) $X \geq 8$ and $Y \geq 6$ (e) $X + 7Y > 50$

V.25 A solution of the inequality $|x - 3| + |x + 2| \leq 6$ is

(a) $-5/2 \leq x \leq 7/2$ (b) $x \geq 6$ or $x \leq 1$ (c) $x \geq 3$ or $x \leq 2$ (d) $2 \leq x \leq 3$ (e) $x \geq 3/2$ or $x \leq -1/2$

VI.30 If $S = a - b + c - d$, where $a > b > c > d > 0$, then which of the statements (I) $S < a$ (II) $S > d$ (III) $S > 0$ are necessarily true?

(a) III only (b) I and II only (c) I and III only
(d) all (e) none

VII.18 If $0 < x < 1$ and $S = 1 + x + x^2 + x^3$ then which of the statements (I) $S > 1/x$ (II) $S < 1/(1 - x)$ (III) $S < 7/(1 + x)$ must be true?

(a) I,III only (b) II only (c) III only (d) none
(e) all three

IX.13 If $a/b < c/d$. where a,b,c,d are positive numbers, then what number must be between a/b and c/d?

(a) $(c/d - a/b)/2$ (b) $(a + c)/2(b + d)$
(c) $(a + d)/2(b + c)$ (d) $(a + 3c)/(b + 3d)$
(e) $(a + 2d)/(b + 2c)$

IX.28 Given three real numbers x,y,z, if $x + y - z$ is a negative number large in magnitude, $x - y - z$ is a very small positive number, and $x + y + z = 1$ then

(a) $y > x > 0$ (b) $x > 0 > y$ (c) $z > y > x$
(d) $z > y > 0$ (e) $z > 0 > x$

X.22 If $0 < x < y < z < 1$ then which of the following must be true: (I) $x^2 < y^3$ (II) $xz < y^3$ (III) $x + y < 2z$ (IV) $xyz < z - x$? (a) III and IV (b) I,II, and IV (c) II and III (d) I and II (e) I and III.