GEMINI TOPIC TEST – CALCULUS LEVEL
2000 MU ALPHA THETA NATIONAL CONVENTION

1. There exists a function f such that $f(x)+2f(x^{-1})=3x^2$. Find $\int_1^2 f(x)dx$.

   A.1 B.1/3 C.-4/3 D.-2/3 E. NOTA

2. Please Evaluate: $\int_{-\infty}^{+\infty} e^{-x^2/2}dx$

   A. $\sqrt{\pi}$ B. undefined C. $\sqrt{\frac{2}{\pi}}$ D. $\sqrt{2\pi}$ E. NOTA

3. A circle of radius 1 m rolls around the circumference of a circle of radius 4 m. The epicycloid traced by a point on the circumference of the smaller circle is described by
   $x = 5\cos(t) - \cos(5t)$ $y = 5\sin(t) - \sin(5t)$ for $0 \le t \le 2\pi$
   Find the distance traveled by the point in one complete trip around the larger circle.

   A.40 B. 10π C. 40π D. 20π E. NOTA

4. Let A equal the number of edges a regular solid with 12 faces and 16 vertices.
   Let B equal the volume of a rectangular solid if the areas of 3 faces are 12, 36, and 48
   Find $\frac{A \times B}{lcm(A,B)}$

   A.8 B.4 C.2 D.1 E. NOTA

5. Let A equal the area of the triangle with vertices at (2,-1,0), (-1,2,1), and (0,1,-1).
   Let B equal the volume of the pyramid with vertices at (2,1,-1), (3,-2,3), (-2,2,4), and (3,-3,2).
   Find $\lfloor A+B \rfloor$, where $\lfloor B \rfloor$ represents the greatest integer function.

   A.10 B.8 C. $6\sqrt{3}$ D.9 E. NOTA

6. Determine b: $1543_{b+1} \times 2153_{b-1} = 3352751_b$ (b represents the number base)

   [illegible] [illegible] C.7 D.8 E. NOTA

7. **Two poles of 18 and 22 feet are planted in the ground, mutually parallel. A wire is drawn between the top of each pole connecting it to the bottom of the opposite pole.** The height of the intersection of the two poles from the ground can be represented by a/b where a, b are relatively prime positive integers. What is the value of $\log_b (a+1)^b$?

A. 20 B. 1024 C.27 D.64 E. NOTA

8. Find a solution (a,b,c) to the following set of equations:

$$2a + 3b + c = 11$$
$$6ab + 2ac + 3bc = 24$$
$$abc = -6$$

A.(2,3,-2) B.(1,-3,2) C. (3,2,-1) D.(-3,2,1) E. NOTA

9. It is possible to expand the expression $(a+b+c+d)^{10}$ to find the coefficient of each term. The coefficient of the term $a^2b^2c^4d^2$ is a rather large number. What is the sum of the digits in this coefficient?

A.5 B.18 C.27 D.16 E. NOTA

10. Let C be the arc of the circle $x^2 + y^2 = 9$ from the point (3,0) to $(3/2\ ,\ 3\sqrt{3}/2)$. Using parametric equations, find the surface area formed by revolving C about the x-axis.

A. $3\pi\sqrt{3}$ B. 6π C. $6\pi\sqrt{3}$ D. 9π E. NOTA

11. In triangle AJO, a=10 m. and A = 60°. What is the area of the circumscribing circle?

A. $100\pi/3$ B. 50π C. $50\pi/3$ D. 100π E. NOTA

12. Find the area of the loop enclosed by the strophoid $r = \sec\theta - 2\cos\theta$, such that $-\pi/2 \le \theta \le \pi/2$.

A. $\pi/2$ B. $4-\dfrac{\pi}{2}$ C. $2-\dfrac{\pi}{2}$ D. $\dfrac{\pi}{2}+2$ E. NOTA

13. When you evaluate the definite integral $\int_{-1}^{2} x(5^{-x^2})dx$, you get an answer in the form $a/(b\ln c)$. When reduced to lowest terms, what is the value of $a+b+c$?

14. A building is to be supported with a linear support brace that must pass over a separate parallel wall, 16 ft from the foot of the building. The wall is 8 feet high. Find the length of the shortest brace that can be used to complete the task. (round to the nearest hundredth of a foot)

A. 33.30 B.33.94 C. 19.58 D. 23.54 E. NOTA

15. Find the area of the surface formed by revolving the graph of $r = \cos\theta$ about the line $\theta=\pi/2$.

A. $\pi^2/2$ B. $\pi/2$ C. 2π D. π^2 E. NOTA

16. The center of mass of a planar lamina $0 \le y \le 4 - x^2$ with density function $\rho(x,y) = y$ (where $\rho(x,y)$ has dimension $mass/unit^2$) is quite easy to find. Using symmetry and a double integral, you can get a fraction in the form a/b where a,b are relatively prime positive integers. What is a+b?

A. (0,16/7) B.(0, 12/7) C. (0,8/7) D. (0,6/7) E. NOTA

17. Given that n=50, find a value for $S = \binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots + n\binom{n}{n}$.

A. $50(2^{50})$ B. $50(2^{51})$ C. $50(2^{49})$ D. $50(2^{51}) - 1$ E. NOTA

18. Find the value of the x^3 term in the Maclauren polynomial of the function defined by $\int_0^x \frac{\ln(t+1)}{t} dt$ evaluated at x=2.

A.4/27 B. 2/27 C. 4/9 D.8/3 E. NOTA

19. The expression $\frac{2}{3} \times \sum_{n=1}^{\infty} \sum_{m=n}^{\infty} \frac{1}{3^m}$ can be reduced to a single fraction a/b where a, b are relatively prime positive integers. Find the value of $(a - 2^b)(e^{i\pi})$.

A. -3 B.3 C. 13 D. -13 E. NOTA

20. An inverted conical tank contains diesel fuel weighing 1.3 lbs./$in.^3$ The tank has a diameter of 8 ft. and a depth of 6 ft. How much work is required to pump all of the fuel out of the tank to a position 2 ft. above the top of the tank (round your answer to the nearest foot-pound)

21. You have probably never heard of Vandermonde's identity. It states that $\sum_{k=0}^{n}\left(\binom{m}{k}\binom{n}{n-k}\right)=\binom{m+n}{n}$. However, you can use it to easily solve for S, given that $S=\sum_{k=0}^{25}\binom{25}{k}^2$. How many digits are there in the number S?

A. 15 B. 30 C. 20 D. 25 E. NOTA

22. The expression $\left(\frac{e^{ix}-e^{-ix}}{e^{ix}+e^{-ix}}\right)^2+1$ can be simplified to:

A. $e^{2ix}+e^{-2ix}$ B. $e^{4ix}-e^{-4ix}$ C. $\tan x$ D. $\sec^2 x$ E. NOTA

23. Imagine that there exists a function such that $f(x+2t)-f(x-2t)=8xt$. Find the derivative of f(x) where x=1/2. Note that $-2\pi \le t \le 2\pi$.

A. 1/2 B. $\sqrt{3}/2$ C. 1 D. $\pi/3$ E. NOTA

24. A sphere with equation $x^2-2x-4y+y^2+z^2+6z=2$ is drilled into by a perfect cylinder. This very long cylinder of unit radius drills completely through the sphere such that its central axis is coincident with the center of the sphere. After the drill goes completely through the sphere, removing maximum volume, what is remaining volume of the sphere? (round to the nearest integer)

A. 234 B. 34 C. 244 D. 24 E. NOTA

25. I can take the matrix $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ (which contains a secret message) and right multiply by a coding matrix, J, to produce an encrypted matrix $\begin{bmatrix} 81 & 79 \\ 53 & 87 \end{bmatrix}$. Now that you know what matrix J is, use it to encode matrix A=$\begin{bmatrix} 9 & 12 \\ -15 & 22 \end{bmatrix}$. What is the sum of the elements of the resulting matrix?

A. 300 B. 1240 C. 615 D. 952 E. NOTA

26. Six men sit down at a long table with seven seats, completely ignoring their nametags. In how many ways can these six men be arranged so that no man is sitting at a seat with his correct nametag?

A. 2520 B. 1854 C. 5040 D. 2624 E. NOTA

27. Let A= the sum of the coefficients of the polynomial of least degree (leading coefficient of 1) that passes through the points (-2,54), (5,23), and (2,-10)
Let B= the volume enclosed when the region bounded by the equations $y = x+2$ and $y = 4 - x^2$ is revolved about $y = \frac{2x}{5} - 4$.

Find A+ $\frac{B\sqrt{29}}{\pi}$

A. -2 B. 5 C. 12 D. 57 E. NOTA

28. Find the value of $\sum_{k=0}^{n} F_k$ given that n equals the number of terms in the reduced expansion of the expression $(2a^2 + 3b - 4\sqrt{d})^4$. (Note: F_k is defined to be the kth Fibonacci number with $F_0 = F_1 = 1$)

A. 3287 B. 2584 C. 2031 D. 2583 E. NOTA

29. Have you ever tried to find the arc length of an ellipse? Well, it has been proven that there is no combination of elementary functions that describe the arc length of an arbitrary ellipse. However, there exist many numerical methods to approximate it. Which of the following answers below gives the best approximation of the arc length of the ellipse $4x^2 + 54y + 9y^2 - 16x + 61 = 0$? (round to the nearest hundredth)

A. 7.36 B. 14.72 C. 18.85 D. 9.43 E. NOTA

30. Jason has a cube of wood. He slices off one vertex of this cube such that he has a triangular pyramid with three edges converging at mutual right angles. The lengths of the three sides that converge to form this right angle are 3, 4, and 6 units. Jason, being a clever fellow, thinks an extension of Pythagoras's theorem into 3-space might help him find the area of the largest triangular face on his newly cut pyramid. Can you find this area?

A. $\sqrt{261}$ B. $\sqrt{61}$ C. $\sqrt{307}$ D. $\sqrt{255}$ E. NOTA