

## Recursion

- 1 Find  $x$  given the following:  $x = \sqrt{30 - \sqrt{30 - \sqrt{30 - \sqrt{30 - \dots}}}}$
- A. -5      B. 5      C.  $\{-5, 5\}$   
D.  $\emptyset$       E. NOTA
- 2 Evaluate  $\frac{1}{1 \cdot 2 \cdot 3} + \frac{2}{2 \cdot 3 \cdot 4} + \frac{3}{3 \cdot 4 \cdot 5} + \dots + \frac{98}{98 \cdot 99 \cdot 100}$ . Express your answer as a fraction in simplest form  
 $\frac{b}{a}$ . Find  $a+b$
- A. 149      B. 24749      C. 14851  
D. 194      E. NOTA
- 3 Express the number  $7.\overline{47}$  as a fraction in simplest form. Now give the reciprocal of the ratio of the denominator to the numerator.
- A.  $\frac{90}{673}$       B.  $\frac{100}{747}$       C.  $\frac{747}{100}$   
D.  $\frac{673}{90}$       E. NOTA
- 4  $\sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20 + \dots}}}}$
- A. -5      B. 5      C. -4  
D. 4      E. NOTA
- 5 If the table below defines certain values of a function  $f$ , then what is  $f(f(f(f(5))))$ ?
- |        |   |   |   |   |   |
|--------|---|---|---|---|---|
| $x$    | 1 | 2 | 3 | 4 | 5 |
| $f(x)$ | 4 | 1 | 4 | 0 | 2 |
- A. 0      B. 1      C. 2  
D. 4      E. NOTA
- 6 Given:  $f(x) = x^3$   
 $g(x) = x + 1$   
Find:  $(f \circ g)(2)$
- A. -2      B. 3      C. 9  
D. 27      E. NOTA
- 7 What is the 5<sup>th</sup> term of an arithmetic sequence with a common difference of 5 and first term of 2?
- A. 13      B. 15      C. 22  
D. 27      E. NOTA

# Recursion

- If  $k(x) = f\left(g\left(h\left(g\left(f\left(h\left(g\left(g\left(h(x)\right)\right)\right)\right)\right)\right)\right)$  and  
 $q(x) = g\left(f\left(f\left(f\left(h\left(g\left(h\left(g\left(h(x)\right)\right)\right)\right)\right)\right)\right)$ , then use the table below to find the value of  $q(k(4))$

$x$	$f(x)$	$x$	$g(x)$	$x$	$h(x)$
1	11	2	5	1	7
2	3	4	13	3	4
4	13	6	17	4	13
7	3	9	2	7	3
9	27	11	2	10	17
11	4	13	27	12	4
13	6	17	4	19	2
27	9	27	12	27	11

- 9 Given that  $f(x+1)f(x) = x$  for all positive values of  $x$ .  $f(2001)f(2004)$  can be written in the form  $\frac{a(a+2)}{(a+1)}$ . What is the units digit of  $a$ ?

A. 1      B. 2      C. 3  
 D. 4      E. NOTA

10 Find the sum of all  $x$  which satisfy the equation  $x = \sqrt{20 - \sqrt{20 - \sqrt{20 - \dots}}}$

A. 5      B. -1      C. -5  
 D.  $4 - 5i$       E. NOTA

11 Given:  $f(x)$  is a one-to-one function,  $f(x) = -f(-x)$ , and  $g(x) = g(-x)$ . Find  $g(f^{-1}(-5))$   
 $f(2) = 5$      $g(3) = -3$   
 $f(3) = 2\pi$      $g(-2) = 3$

A. -2      B. 5      C. 3  
 D. -3      E. NOTA

12  $x = \sqrt{5 - 4\sqrt{5 - 4\sqrt{5 - 4\sqrt{\dots}}}}$  What is the sum of all possible values of  $x$ ?

A. -3      B. 1      C. 4  
 D. 5      E. NOTA

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- 13  $4 - \frac{4}{4 - \frac{4}{4 - \dots}}$   
 A. 1      B. 2      C.  $3.\bar{8}$   
 D. 16      E. NOTA

14 Find the unit's digit of  $1 + 9 + 9^2 + 9^3 + \dots + 9^{n-1} + \dots + 9^{1988} + 9^{1989}$   
 A. 0      B. 1      C. 7  
 D. 9      E. NOTA

15 If  $f$  is a linear function and  $f(f(x)) = 2x + 4$ , find the positive value of  $f(1)$   
 A.  $\sqrt{2}$       B. 4      C.  $4(\sqrt{2} - 1)$   
 D.  $5\sqrt{2} - 4$       E. NOTA

16 Evaluate as a common fraction:  
 $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\frac{1}{4}}}}$   
 A.  $\frac{5}{6}$       B.  $\frac{11}{6}$       C. 2      D.  $\frac{11}{5}$       E. NOTA

17  $x = \sqrt{210 + \sqrt{210 + \sqrt{210 + \dots}}}$   
 A. 14      B. 15      C. 224  
 D. 225      E. NOTA

18 If  $f(x) = \frac{1}{2}x + 1$  and  $g(x) = -\frac{2}{3}x - 4$ , then the slope of  $f(g(x))$  is ?.  
 (A)  $-\frac{1}{3}$       (B)  $-\frac{1}{6}$       (C)  $\frac{7}{6}$       (D) -3      (E) nota

## Recursion

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- $\sqrt{42 - \sqrt{42 - \sqrt{42 - \dots}}} = N$ . Find the value of N.
- (A) 5      (B) 6      (C) 7      (D)  $\sqrt{42}$       (E) nota

20

Find the value of  $\frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}}$

- A.  $\frac{3}{4}$       B.  $\frac{4}{5}$       C.  $\frac{5}{6}$   
D. 1      E. NOTA

21 The functions  $f$  and  $g$  are inverses, both with domains of all real numbers. If  $f(2) = 5$  and  $g(3) = 6$ , then  $g(5) + f(6) =$

- A. -11      B. 5      C. 0  
D. cannot be determined      E. NOTA

22 If  $f(x) = 9x + 6$  and  $g(x) = 3x^2$  then which is equal to  $g(f(1))$ ?

- A. 2025      B. 675      C. 45  
D. 33      E. NOTA

23 If  $f(x+2) = x^2 + 4x$ , then which is equal to  $f(5)$ ?

- A. 45      B. 43      C. 21  
D. 9      E. NOTA

24 Find the absolute value of the difference of the solutions for the equation

$$(x+3)^2 + (x+3) - 2 = 0$$

- A. 0      B. 1      C. 3  
D. 7      E. NOTA

25 When simplified, the product

$$\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \cdots \left(1 - \frac{1}{n}\right)$$
 equals

- A.  $\frac{1}{n}$       B.  $\frac{2}{n}$       C.  $\frac{2(n-1)}{n}$   
D.  $\frac{2}{n(n+1)}$       E. NOTA

26 If  $f(x) = x^2 - 4$  and  $g(x) = x^2 - 9$ , what is the value of  $f(g(f(\sqrt{7})))$ ?

- A. -4      B. -2      C. 0  
D. 4      E. NOTA

# Recursion

- 27 Find the exact value of the expression  $\sqrt{2+3\sqrt{2+3\sqrt{2+\dots}}}$

A. 1      B.  $\frac{3+\sqrt{17}}{2}$       C. 2

D.  $2+3\sqrt{5}$       E. NOTA

28 If  $f(x) = \sqrt[3]{x^2 - 2x}$ , what is the positive value of the inverse of  $f$  at  $x = 2$ ?

A. 2      B. 4      C. 8

D. 10      E. NOTA

29 Given that  $f(x)$  is the inverse of  $g(x)$ , if  $f(x) = \frac{3x}{x+2}$ , what is  $g(1) + g(2)$ ?

A. 2.5      B. 5      C. 6

D. 7      E. NOTA

30 If  $2f(x) = xf(x+1) + 6$ , what is  $f(-4)$ ?

A. 3      B.  $\frac{3}{2}$       C.  $\frac{3}{4}$

D.  $-\frac{3}{4}$       E. NOTA

## Recursion

### ANSWERS

1. B
2. A
3. D
4. B
5. A
6. D
7. C
8. C
9. A 
$$\frac{f(2004)f(2003) \cdot f(2002)f(2001)}{f(2003)f(2002)} = \frac{2001 \cdot 2003}{2002}$$
10. E 4
11. C
12. B
13. B
14. A
15. E  $5\sqrt{2} - 4$
16. B
17. B
18. A
19. B
20. B
21. B
22. B
23. C
24. C
25. B
26. A
27. B
28. B
29. B
30. B