Middleton 1/10/04

Algebra II Team (Sponsor Copy)

1. If $f(x)=5x+2$, let A be the value of x for which $f(x)=f(1)+3$
let B be the value of x for which $f(3)=x+6$, let C be the value of $f^{-1}(5)$ for f^{-1}, the inverse of f. Give the value of 10(A+B+C).

2. If $f(x)=16^x$ then let R be the value of $f\left(\frac{3}{4}\right)$, let S be the value of x for which $f(x)=\frac{1}{4}$, let T be the value of x for which $f(x)=4^{1-6x}$. Give the value of $8(R+S+T)$.

3. The depth of water (in feet) in pool A is given by the function $d(t)=3t+1$ for $0\le t\le 5$.
The depth of water (in feet) in pool B is given by $h(t)=18-t$ for $0\le t\le 5$.
Let M be the value of t when the depth of water in pool B is twice that of the depth of water in pool A.
Let N be the absolute difference in depths of water in the two pools at $t=2$.
Let Q be the value of t when the depth of water in pool A is 4 less feet than the depth of water in pool B.
Give the value of 7M+8Q+N.

4. Point A has coordinates (8, 2) and point B has coordinates (6, 12).
If the perpendicular bisector of the segment $\overline{AB}$ has equation $Dx+Ey=F$, for coefficients D, E, and F relatively prime integers, and $D>0$, then give the value of $D+E+F$.

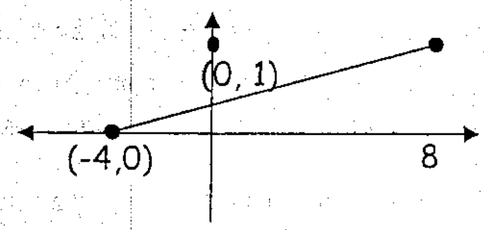
5. Juan played video games and won 150 and lost 25. No ties are possible. Let P be the number of additional games he will now have to play and win (no losses, just consecutive wins) in order to have won 90% of all games played, including the original 175 games.
If he continues playing (after the original 175 games) and wins 9 of every 10 games played from that point on, let Q be the number of additional games he would have to play in order to have won 88% of all games played (including the original 175 games).
Give the value of P+Q.

6. George has taken four tests and earned 50%, 80%, 70%, and 90%.
Let A be the percent he must earn on his next (fifth) test in order to have an average (mean) of 75%.
Let B be the average of his fifth and sixth tests if it is known that the average (mean) of the six tests is 78%.
Let C be the absolute difference between the current four-test average and the value of the average if the lowest test were dropped of the four.
Give the value of A + B + 2C.

7. Consider the system of linear equations $\begin{matrix} x+By=C \\ 3x+4y=12 \end{matrix}$. If the solution to the system is (4, 0) and the lines are perpendicular, then give the value of $B+C$.

Middleton 1/10/04

8. The segment defined by $y = f(x)$ for $x - 4y = -4$ is graphed over the domain [-4, 8] as shown.
Let R be the minimum y-value of the graph of $y = f(|x|)$, and let S be the y-value of the graph of $y = -f(x)$.
Give the value of R+S.



9. $f(x) = i\sqrt{x}$ for $i = \sqrt{-1}$
Let S be set of values of x for which the values of $f(x)$ are real.
$g(x) = |2 - x|$
Let T be the set of values of x for which $g(x) < 5$.
How many INTEGERS are in the intersection of the sets S and T ?

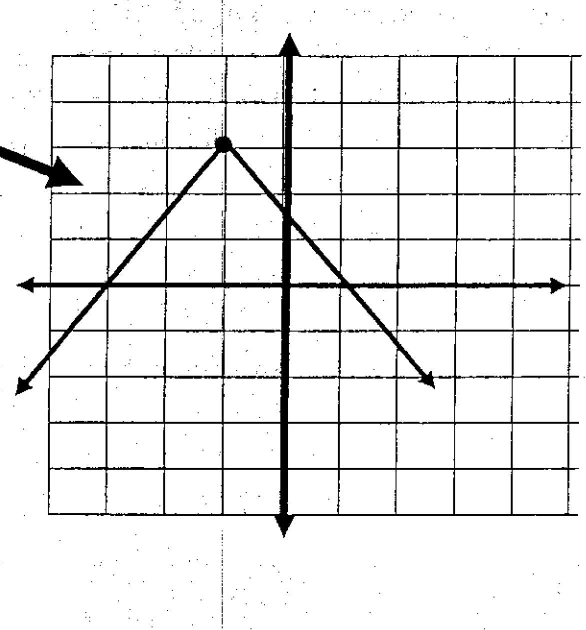
10. For $f(x) = 3x - 6$
Let the point (A, 0) be the point of the x-intercept of the graph of f.
Let the point (0, B) be the point of the y-intercept of the graph of the inverse of f, $f^{-1}(x)$.
Let the point (C, D) be the intersection of the graphs of f and the inverse of f, $f^{-1}(x)$.
Give the value of A + B + C + D.

11. The line L_1 has equation $\frac{x}{A} + \frac{y}{B} = 1$ with constants A and B such that $2A + B = 5$ and $6A - B = -1$. Let S be the slope of line L_1. Give the value of $10(A + B + S)$.

12. Let A be the largest integral value of x for which $\sqrt{6 - \sqrt{6 - x}}$ gives a result which is a positive integer.
Let B be the smallest integral value of y for which $|4y - 2| < 10$. Give the value of A^B.

13. A vat contains 30 gallons of a solution which is 10% saline.
Let R be the number of gallons of pure saline which must be added to the vat in order to make the solution 40% saline.
Let T be the number of gallons of a 30% saline solution which must be added to the 30 original gallons in order to make the solution 25% saline.
Give the value of R+T.

14. The graph of $f(x) = a|x + b| + c$ is shown.
Increments on the axes shown are 1.
The roots of the graph are -3 and 1.
The maximum point on the graph is (-1, 3). Give the value of $2(a + b + c)$



15. Let $A = \sqrt{20 - \sqrt{20 - \sqrt{20 - \ldots}}}$
Let B be the sum of the roots of the graph of $y = 12 - x^2$, and let C be the sum of the first whole number and the greatest negative integer. Find the value of $-ABC$.