Mathematical Vocabulary: Fixers of Knowledge or Points of Exploration?

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This paper examines the idea that in mathematics education it is important to wean pupils off the use of informal everyday language and to privilege the use of formal technical vocabulary. I will first make some observations on the use of formal and informal language in the Dimensions transcript. The main focus of the next part of the discussion is on the complexities in establishing core and non-core vocabulary meaning and the need to use words to represent established meaning/s as well as to create new ones. After that I will draw on research in mathematics education to show that informal and formal language (including technical vocabulary) is used in various combinations and that pupils can, indeed need to, use informal language productively to explore concepts represented by technical vocabulary.

Keywords: mathematical vocabulary, vocabulary learning, lexical development, semantisation, participatory discourse, mean-making

Assumptions and Questions

The advice given to teachers on mathematical vocabulary (see introduction[1], this volume, Appendix 1) raises a number of questions on language and learning, and on the relationship between them. In this paper I will focus on one specific issue: the use of language to talk about technical mathematical vocabulary in the classroom. I will address this issue primarily from the perspective of a language educator working in linguistically diverse classrooms. Therefore I will draw on both first and second language vocabulary learning literature where appropriate.

It would be useful to start the discussion by foregrounding one of the central assumptions of the advisory text that is relevant to this discussion: in a mathematics lesson it is important to wean pupils off everyday language and to use technical mathematics language in a planned and systematic way.

> Teachers often use informal everyday language in mathematics lessons before or alongside technical mathematical vocabulary. Although this can help children to grasp the meaning of different words and phrases, you will find that a structured approach to the teaching and learning of vocabulary is essential if children are to move on and begin using the correct mathematical terminology as soon as possible. (DfEE, 1999: 2)

The technical and specialist[2] use of language for specific purposes can be interpreted in two very different ways: (A) technical language as a sign of expertise and valued knowledge (positive evaluation) and (B) technical language as unnecessary jargon (negative evaluation). Interpretation (A) is usually associated with the idea that knowing technical language *is* part of having technical knowledge and expertise. Interpretation (B) usually implies that no (worth-

0950-0782/05/02 127-09 $20.00/0
LANGUAGE AND EDUCATION

Vol. 19, No. 2, 2005

while) technical knowledge underlies the technical language. Both interpretations are underpinned by an implicit acknowledgement that the use of technical language is a form of meaning making and meaning representation. The evaluation given to specific instances of such language use will depend on speakers' interests and preferences. The advisory text on mathematical vocabulary clearly positions itself as belonging to the first category. Perhaps this is unsurprising since any formal teaching advice on a curriculum subject, by virtue of being so presented, has to claim technical and specialist knowledge status. What is more interesting though is that the text cited above appears to suggest that in order 'to move on' in mathematics, one needs to be rid of informal everyday language expressions and to use 'correct mathematical terminology'.

This paper is not concerned with the validity of the argument that pupils should acquire technical mathematical vocabulary. It is assumed that the language of mathematics, just like other curriculum subjects, comprises technical vocabulary and expressions, and ordinary informal language. From a language education point of view an important issue is: what does the learning of technical mathematics vocabulary and its associated concept/s entail? A related and slightly broader question is: what is the relationship between technical vocabulary and informal everyday language? I will eschew the issues concerning the specific relationship between language-based discourse and learning and proceed on the epistemological basis that language and meaning are intertwined in thought and social interaction (for a discussion see Cruickshank, ms; Sfard *et al*., 1998; Sierpinska, 1998).

Meaning Making Through Participatory Discourse

The Year 5 classroom transcript (see introduction, this volume, Appendix II) shows interesting shifts in language registers and classroom interaction patterns. Once the teachers (T and RB) and the pupils passed through the more formal teacher-led Initiation (I)–Response (R)–Feedback (F) routine (turns 1–10), the more informal everyday mode of language was used in the negotiation and making of mathematical meanings. The use of language from turn 12 is informal in two senses. Firstly, the teacher–pupil exchanges do not follow the usual teacher-led I-R-F sequence. The interaction between the two teachers and the pupils is much more contingent, involving a larger number of turns by pupils; teacher 15 turns: teacher-researcher (RB) 5 turns: pupils 25 turns. The teacher returns to a more didactic move in turn 32 briefly trying to address the notions of 'circumference, diameters and radius', but in other turns we can see more open and contingent engagement. For instance, in turn 15 the teacher-researcher chimes in with pupil V and encourages this pupil to complete his self-nominated turn. In turns 26 and 30 the teacher asks clarification questions which are in effect responses to the information offered by pupils. In other words, the pupils are active partners in shaping and extending the exchanges.

Secondly, a good number of the actual language expressions used by participants are also informal. Apart from the topic-related technical terms such as dimension and sphere, most of the insightful mathematical points are expressed through informal everyday language. For instance, in turn 46, pupil K, who has up to this moment not actively participated in the discussion, says, 'There's no such thing as a one dimensional shape coz a line is kind of like a rectangle filled in'. The informal description 'like a rectangle filled in' captures succinctly the idea that even a line on a page or on a whiteboard has more than one dimension. Other informal expressions such as turn 54 ' . . . has just a tiny tiny tiny' (accompanied by gesture) and turn 55 'Very thin' are clearly on-task and oriented towards the topic in hand. From a classroom discourse analysis perspective there is a reasonable case to suggest that the informal and relatively open nature of the exchanges has enabled at least some of the pupils involved to achieve some insight into the complex meaning of the concept represented by the term 'dimension', or more accurately the terms 'one-dimension', 'two-dimension' and 'three-dimension'.

Meaning or Meanings?

From the point of view of language learning we can explore this particular episode of 'dimension' learning as a particular instance of vocabulary learning in a more general sense. In the following discussion two assumptions are made – that vocabulary items in a natural language such as English are taken to be represented or manifested by words and that words correspond with real world entities such as 'restaurant' and abstract concepts such as 'dimension'. Learning a word is taken to mean at least the learning of its formal features (e.g. sound and written representation) and its meanings. The main attention of this discussion is on the meaning aspect of word learning. Two questions seem relevant here: does learning an item of vocabulary mean learning its clear and unambiguous meaning; and what else does learning vocabulary entail? I will look at these two questions in turn.

The answer to the question of whether learning an item of vocabulary means learning its clear and unambiguous meaning depends largely on whether vocabulary items have clear meanings in the first place. Here we run into the core versus non-core meaning issue in the field of vocabulary studies[3]. Some would argue that words have fixed core meanings; for instance the word 'cat' or 'dog' can be said to have a fixed core meaning based on a mental image. However, there is a difficulty with this view because 'cat-ness', for instance, may have a number of aspects or characteristics such as 'sleeping cat', 'young cat', 'walking cat', 'black cat' and so on. Given this possibly open set of characteristics it is not clear how one may define core 'cat-ness'.

One possible way out of this dilemma is to adopt the 'conditions of criteriality' or 'a checklist theory', as Aitchison (1994: 43) calls it. In essence the checklist theory states that to define the meaning of a word is to identify a set of necessary and sufficient conditions for the thing or concept represented by that word. Aitchison (1994) illustrates this with the word 'square' which can be said to have four necessary conditions:

1 a closed, flat figure
2 having four sides
3 all sides are equal in length
4 all interior angles are equal

Each of these conditions is, by itself, necessary in order to for something to

> be a square, and, when combined, they are sufficient to define and identify a square, and only a square.

At first glance this approach appears to offer a way forward. However, there is a problem with working out which condition/s should become criterial. For example, a square is defined as:

> a figure with 4 straight equal sides forming 4 right angles

by the Longman English-Chinese Dictionary of Contemporary English (Longman, 1988):

> 7. a. *Geom.* A plane rectilinear and rectangular figure with four equal sides; . . . a rectangle with unequal sides . . .

by the Oxford English Dictionary:[4] and

> A regular quadrilateral.
> It is a rectangle whose sides are all the same length.
> It has four lines of symmetry, and rotational symmetry of order 4.
> Opposite sides are parallel, and all four angles are right angles (90°).
> Its diagonal bisect each other at right angles, as well as bisecting the angles.
> (Definition level 1), by the thesaurus.maths.org[5]

Which of these definitions should one adopt? The answer may depend on what one wishes to do with the definition or with the word. But a relevant point for this discussion is that this example shows that even with a word such as 'square', which is generally regarded as representing a precise concept, it is by no means simple to decide on its criterial conditions. The difficulty with more everyday words such as 'restaurant' can therefore be easily seen. Does a restaurant have to be an indoors establishment? Does it have to provide table service? Does it have to offer a menu of dishes? Does it have to be of a particular size? These and many other similar questions pertaining to the characteristics of 'restaurantness' can be posed. It may well be that many (or even all) of these characteristics are core but they do not have to co-occur at the same time. A moment's reflection will suggest that there are many facets to restaurants and that different restaurants may display different sets of characteristics. 'Restaurant-ness' is thus fuzzy and the concept itself may carry a good deal of core and non-core meanings in various combinations. Indeed most words appear to have variable combinations of core and non-core meanings. Yet most people seem to be able to use and understand the word 'restaurant' and other similarly non-core words without great trouble. This suggests that for a word to be understood and used it isn't necessary to know a tightly defined pinned down meaning.

Quite clearly knowing a word means knowing more than its core meaning (where core meaning can be identified). Very often we need the facility and the possibility of non-core meanings to help us make sense of what others say and to enable us to represent and create meaning. The capacity of words to carry non-core meanings seems to be an enabling facility in our efforts to describe, explore and analyse perceptions and experiences. To be able to use the word 'restaurant' when talking about an establishment that is open-air, operates from 10:00 pm onwards, has no serving staff and requires patrons to select their own ingredients from the food counter for the kitchen staff is to have the freedom to extend the conventional meaning of 'restaurant' and to use the term to describe new or different concepts and practices associated with the existing core and non-core meanings. We can see this kind of use of language as part of reasoning invoking generalisation and expansion of pre-established meanings. So, seen in this light, words don't just represent what we claim to know already, they also allow us to make observations and to formulate novel meanings within a negotiated range of acceptable/accepted possibilities or limits.[6]

Learning Words, Learning Mathematics, Learning Processes

Schmitt (2000) observes that learning a word is an incremental process. On first verbal encounter with a word, a person might remember the sound or some features of the sound, e.g. the number of syllables. If the first exposure is in the written form, a person might remember some letters of the word. In terms of meaning representation, only one particular sense (of core or non-core meanings) is likely to be gained – the sense in which the word might have been meant on that occasion. These basic formal and semantic features of a word are built up and consolidated with further exposures and use. 'But it will probably be relatively late in the acquisition process before a person develops intuition about the word's frequency, register constraints, and collocational behaviour' (Schmitt, 2000: 118) because these linguistic properties are less easily noticed in actual instances of use. The process perspective adopted by Schmitt on acquiring the full range of word meaning is of particular relevance at this point of the discussion. It is consistent with Aitchison's (1994) notion of 'packaging' and Henriksen's (1999: 308) term 'semantization process':

> . . . an ongoing and simultaneous process of developing semantic (i.e. definitional, referential, or extensional links) understanding of a word and working out its semantic relation to other lexical items in the complex structure of the mental lexicon . . .

All of this suggests that learning a word is not a simple and straightforward matter of getting and learning its definitive meaning/s. There is a good deal of fluidity in word meaning which makes it less than helpful to see words as mere carriers of fixed meanings. The learning of a word such as 'restaurant' also involves, among other things, using it as a broad concept to label a range of different configurations of associated meanings and at the same time thinking about these meanings through the broad concept represented by the word itself. As Vygotsky (1986: 213) observed:

> A word might denote at first one object and then become associated with another, just as an overcoat, having changed owners, might remind us first of one person and later of another . . . From primitive generalizations, verbal thought rises to the most abstract concepts.

Does this particular account of word learning have any resonance in the field of mathematics education? It would seem that there is a corpus of work which works with some broadly similar ideas and issues. Monaghan (1999a, b), for example, reports that the term 'diagonal' is used in the materials of a particular (secondary school) mathematics teaching scheme in two senses: as an attribute of shape (a strictly mathematical sense, which we may take to be the core meaning in this discussion) and as a synonym for any oblique line (more everyday sense). He further observes that in these teaching materials the everyday sense is used more prevalently, and that at the lower end of the attainment scales the term is used as a synonym for an oblique line, for instance, to describe a move in a board game with counters, and at a higher level of attainment it is used as a metaphor in a task on mathematical proof. Here we see the issues surrounding core-non-core meanings being played out in a particular set of mathematics pedagogic materials. At a higher level of generalisation Chapman (1997: 166), in a study of spoken language practices in school mathematics, makes a similar observation:

> It is difficult to provide . . . exact categories of language 'types' as the 'hybrid' register of the mathematics classroom changes from lesson to lesson, among various activities and within even very brief interactions. There are multiple and continual shifts back and forth between less and more mathematical language. However, the overall trend typically is towards more mathematical language.

These observations question the idea that we can safely assume that it would be possible for school students to *do* mathematics successfully just with a knowledge of the technical meaning of the subject vocabulary. Furthermore they point to the need for students to explore and learn the range of possible core and non-core meanings in different contexts of use.

In another paper Monaghan (2000) reports a study of a group of secondary (Year 7) students' understanding of quadrilaterals. The particular five-level model of geometric development (the van Hiele model) adopted by Monaghan for his empirical investigation seems to be germane to this discussion. A brief quotation of the first four levels would help us see its relevance:

> Level 0 (Basic level): Visualisation
> . . . Geometric figures . . . are recognised by their shape as a whole, that is, by their physical appearance, not by their parts or properties.
> Level 1: Analysis
> . . . through observation and experimentation students begin to discern the characteristics of figures. These emerging properties are then used to conceptualise classes of shapes . . . Relationships between properties, however, cannot yet be explained by students at this level . . .
> Level 2: Informal deduction
> . . . students can establish the interrelationships of properties both within figures (e.g. in a quadrilateral, opposite sides being parallel necessitates opposite angles being equal) and among figures (a square is a rectangle because it has all the properties of a rectangle). Class inclusion is understood. Definitions are meaningful. Informal arguments can be followed . . .
> Level 3: Deduction
> . . . the significance of deduction is understood. The interrelationship and role of undefined terms, axioms . . . is seen . . . (Crowley, 1987: 1–3, cited in Monaghan, 2000: 181)

If one accepts the validity of the idea of staged development in the learning and understanding of geometric concepts, then something quite close to Schmitt's notion of incremental learning seems to be at work in this model. While it would be unwise to claim, in the absence of any direct evidence, that the cognitive and psycholinguistic processes involved in learning an everyday word such as 'restaurant' and a technical term such as 'quadrilateral' are the same, the common trajectory from undifferentiated recognition to elaborated understanding in both cases do suggest a similar direction of development.

In a discussion on mathematical reasoning Russell (1999: 2) cites a verbatim account of Katie's efforts, a third grade student, to find factor pairs for 120. After working with a partner on paper arrays and a computer game in which they have to fill in arrays with different dimensions, Katie reports to her class:

Katie: Micky and I did three by forty.
Teacher: All right, and did you double-check it? How did you know it would be the same?
Katie: Because, well, actually at first we thought it was three by forty-two, but then on another day . . . it was six by twenty, and we thought, well, if we split six in half and we get three, then we would have to double twenty and we got forty, not forty-two.

A factor pair can be defined abstractly as two numbers that when multiplied together yield a given number. Russell suggests that two thinking processes seem to have occurred in this instance of reasoning, both of which involve making generalisations. First, from her previous work on this topic Katie has learned that a rectangular array is one way of modelling the relationship between a product and one of its factor pairs, and that by maintaining the area of the rectangle but changing its shape, it is possible to generate new factor pairs. Second, the 3×40 answer seems to indicate that Katie is making another generalisation: 'if you halve one number in a factor pair and double the other, you generate a new factor pair that is equivalent to the first pair and therefore equivalent to the same product' (Russell, 1999: 3). Two observations can be made here. First, just as the exchanges and the reasoning in the Dimension episode appear to have been mediated by informal everyday language, Katie's account is expressed through everyday talk. Second, the task to find factor pairs for a particular product compels students to explore possibilities and permutations with reference to a particular product. In this particular instance, it is not enough to simply understand the core meaning of the technical term 'factor pair'; students are expected to think with and through the concept represented by the technical vocabulary. The exploring of the relationship between the two numbers in a factor pair has produced a valued generalisation that goes beyond the initial task.

Mathematical Vocabulary: A Case of Exploring and Fixing Ideas

To summarise some of the key points that have emerged in this discussion we can say that there are at least three related processes involved in vocabulary learning:

- Learning vocabulary, whether in a technical domain or in everyday use, means learning both formal and semantic (core and non-core meanings) features of words in a variety of contexts.

- Learning vocabulary involves thinking with and through the concepts associated with the word/s involved; this means exploring limits and boundaries of word meaning, generalising and extending meaning from one instance to another, and these thinking and negotiating processes are mediated through informal everyday language.
- Learning vocabulary, particularly in terms of its associated concepts and linguistic properties, is an incremental activity; the meanings of an item of vocabulary can develop and expand as part of meaning making.

These observations seem to be consistent with the discourse interaction in our data. The classroom exchanges in the Dimension episode strongly suggest that the teaching of technical vocabulary doesn't need to be exclusively concerned with dealing with fixed meanings. This discussion has shown that when teachers and pupils are engaged in classroom explorations of mathematical concepts and ideas, informal and everyday language can play an important facilitative role. The questioning of and the collaborative talk on the concept of one-dimension by both students and teachers is a good case in point. On this view, the teaching of technical vocabulary should be seen as a pedagogic point of departure for exploring concepts, meaning-making and meaning exchanging, not an end point of learning.

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Notes

1. For details of the texts referred to in this paper, which is one of a set, see the introductory paper this volume, 'Language in the Mathematics Classroom', pp. 97–102).
2. Some writers maintained a distinction between technical and specialist vocabulary, e.g. Monaghan (1999a). For the purpose of this discussion, the term technical vocabulary will be used to represent the use of language in a non-everyday way in mathematics lessons.
3. The distinction between core and non-core meaning has been discussed by a number of writers and contrasting terms such as 'denotation v. connotation' and 'core v. encyclopedic' have been used. For a discussion see Carter and McCarthy (1988, Chap. 2) and Schmitt (2000, Chap. 3).
4. http://dictionary.oed.com/ accessed on 31-08-2003.
5. http://thesaurus.maths.org/mmkb/entry.html?action=entryById&id=6844 accessed on 16-06-2004.
6. The limit or range of acceptability appears to be socially (in the broadest sense of that term) determined and it can shift over time. The contemporary meanings of words such as 'cool' (indicating approval) in the past 30 years are good examples.

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