Ambiguity in the Mathematics Classroom

Richard Barwell
University of Bristol, UK

Mathematics is commonly seen as a discipline with no place for linguistic ambiguity. In this paper, the treatment of ambiguity in two data extracts is critically examined. Analysis draws on two contrasting models of the nature of mathematics and mathematical language. The formal model sees meaning as fixed and relating to language relatively unproblematically. The discursive model sees meaning as situated in and by interaction, and so as shifting and changing as interaction unfolds. Analysis of the extract from the National Numeracy Strategy suggests that it is based on the formal model. This analysis is contrasted with analysis of classroom interaction which reveals how, from a discursive perspective, ambiguity can be seen as a resource for doing mathematics and for learning the language of mathematics.

Keywords: mathematics education, classroom discourse, primary school education, discourse analysis, ambiguity

Introduction

The notion that there can be ambiguity in mathematics is perhaps counter-intuitive. Mathematics is widely seen as a precise language with which to describe aspects of the world. Pimm (1987) summarises this view:

> . . . there are right and wrong answers to everything, together with clear-cut methods to be taught and learnt for finding them. So how can mathematics be discussed when there is no place for opinion, informed or otherwise? While there might be open problems at the frontiers of mathematics, it is all sorted out and written down at the school level. (Pimm, 1987: 47)

This view of mathematics extends to mathematical language: perhaps more than in any other discipline, terms are seen as being precisely defined. Any ambiguity, that is, any possibility of more than one interpretation for a mathematical expression, arises from sloppy use of language rather than any uncertainty in the mathematical ideas. The UK's *National Numeracy Strategy*, I will argue, tends towards this view of the role of ambiguity in mathematics classrooms.

In counter-position to this view, is one which sees mathematics as a discursive, social process. Research in mathematics education allied with this position considers the processes of meaning-making in mathematics classrooms. Brown (1997), for example, who draws on work in post-structuralism, argues:

> Saussure's work has taught us that meaning is not derived from individual terms but is consequential to the play of differences between successive terms in a particular discourse. When applied to mathematical terms we abandon the notion of individual terms having intrinsic meaning but rather see meaning as dependent on the individual construction of mathematical expressions. (Brown, 1997: 75)

0950-0782/05/02 118-09 $20.00/0
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From such a perspective, meaning is seen as subjective, situated and in a state of flux. If mathematical meaning is dependent on individual construction, the possibility of ambiguity is ever present.

In this paper, I illustrate how ambiguity is present in mathematics classroom discourse. Drawing on a discursive analysis of the data informing this set of papers, I show such ambiguity can be an important feature of mathematics classroom discourse, acting as a significant resource for participants. This ambiguity is shown to be in tension with the underlying assumptions concerning the relationship between mathematics and language implicit in the *National Numeracy Strategy* (NNS) (DfEE, 1999, 2000). I will begin, however, by briefly discussing the nature of ambiguity.

Two Views of Language and Mathematics

In this section I contrast two perspectives on the nature of ambiguity, particularly in relation to mathematics. The first I will call the 'formal model', the second the 'discursive model' (see Linell, 1998: 3 for a similar distinction).

The formal model

Ambiguity arises where more than one meaning can reasonably be attributed to a particular word or phrase. Ambiguity is often seen as relating to a dimension of formality (see Gee, 1999: 30–4). 'Informal' interaction is seen as being highly contextualised, with meaning largely implicit. 'Formal' interaction is seen as being more explicit. Academic writing, as an example of formal interaction, is seen as attempting to avoid ambiguity, through, for instance, the definition or exemplification of terms. The distinction between formal and informal arises in work on the nature of mathematical language, and tends to be couched in terms of the 'mathematical' and the 'everyday'. Mathematical language is seen as particularly averse to ambiguity, as Pimm (1987) has argued. Research into the language of the mathematics classroom has highlighted confusion between the mathematics register and everyday language as a particular and problematic source of ambiguity for students (see Monaghan, 1999, 2000; Pimm, 1987; Rowland, 2000). This confusion is not restricted to technical vocabulary items (as discussed by Leung, this volume). Pimm (1984), for example, discusses the ambiguity surrounding the use of 'we' in mathematics classrooms, where 'we' can include: we in this class (*we've found two solutions*); the teacher, who wants the students to join her (*we'll call this x*); a vague sense of 'people who do mathematics' (*we use x and y for unknowns*). Thus, ambiguity arises in mathematics classrooms from the use of everyday language, which is seen as intrinsically more ambiguous than mathematical language, and which creates ambiguities where mathematical and everyday language overlap.

The discursive model

From a discursive perspective, both mathematics and the teaching and learning of mathematics are seen in terms of discursive practice. Linell (1998: 3), for example, argues, 'languages are constructive and constitutive of the ways we act and think in the world' (see also Edwards, 1997). Mathematics, therefore, is constructed through discursive activity, i.e. through the use of spoken, written or symbolic interaction, including the use of gestures and other non-linguistic aspects of interaction. This discursive activity is made up of discursive practices, ways of using linguistic resources to do mathematics or to do the teaching and learning of mathematics. This process must be seen as social in nature; indeed the social nature of interaction, such as the construction and maintenance of identities, power relations and relationships, can be seen as the primary basis for the organisation of interaction (Edwards, 1997; Edwards & Potter, 1992; Linell, 1998). 'Defining' is one example of a set of discursive practices important in mathematics, as discussed by Morgan (this volume). I say *set* of practices, since, as Morgan shows, there are different ways of doing 'defining' and different ways of using definitions. As Morgan also shows, these different practices are closely linked to social issues, such as the construction of different kinds of mathematics for students working at different levels of the same curriculum.

From this discursive perspective, the notion that ambiguity relates to informality cannot be sustained. All words are to some degree 'ambiguous'. All words can be used to imply differing or multiple meanings. Furthermore, meaning arises through interaction, so that it may be seen as developing and changing for participants as interaction proceeds. Indeed it is this flexibility which makes rich interaction possible, infinite subtlety arising from finite means (Edwards, 1999). Gee (1999: 31) contends that the meaning of an utterance is related not only to the words used, but to the broader 'social languages' invoked. The language of a particular mathematics classrooms may be seen as a social language, which may involve the use of varying degrees of mathematical formality. Within mathematics education, Moschkovich (2003) has argued that a rigid identification between formality and mathematical discourse may be counter-productive, since 'informal' or 'everyday' language may nevertheless be used as part of mathematical practice. As an example she contrasts the use of a formal definition by a teacher with students using everyday language to formulate and develop a working definition, both of which are examples of mathematical discursive practices.

In the sections that follow, I examine the two data extracts on which these papers draw. My examination is discursive in nature. I am interested in how ambiguity is accomplished in the two texts. How are notions of ambiguity brought about? How are they used or deployed? What are these 'ambiguity practices' used to do?

Ambiguity in the NNS

The guidance on teachers' use of mathematical language (DfEE, 2000; see introduction[1], this volume, Appendix 1) appears as part of a booklet entitled *Mathematical Vocabulary*, which consists largely of lists of words designed to be suitable for each year-group from Reception to Year 6. In a brief introductory section entitled 'How do children develop their understanding of mathematical vocabulary?', teachers are provided with three paragraphs of advice to support them in their use of language in the mathematics classroom. The main points of this advice are:

- children need support to move on from 'informal' to 'technical' language in mathematics, and from hearing and speaking new vocabulary to reading and writing;

- teachers should ascertain the extent of children's mathematical vocabulary and the depth of their understanding;
- the introduction of new vocabulary is important and teachers should structure carefully how they do this.

The underlying model on which the guidance appears to be based sees mathematical meaning as separate from language and views technical terms as being used to convey precise mathematical meanings. This model is illustrated by the following extract:

> You need to plan the introduction of new words in a suitable context . . . Explain their meanings carefully and rehearse them several times . . . sort out any ambiguities or misconceptions your pupils may have through a range of . . . questions. (DfEE, 2000: 2)

From this perspective, words wait in the dictionary (or vocabulary booklet) until the teacher judges they can be brought out. Their arrival is marked by careful explanations of their meanings. There is an assumption, therefore, that words have neat, universal meanings that can be carefully explained. These meanings, moreover, are to be imparted, or transmitted, to students. While ambiguities may arise, these are the result of misunderstandings on the part of students. Furthermore, such ambiguities are likely to be between technical and everyday words rather than within the discourse of mathematics itself (Pimm, 1987; Rowland, 2000). Through suitable questioning, teachers can identify and then correct these ambiguities, with the ultimate aim of all students attaching identical, 'correct' meanings to items of mathematical vocabulary. Thus children's and teachers' meanings can be accessed reasonably transparently through language. It is not clear whether these assumptions are related to a view of mathematics as consisting of clear, precise, unambiguous ideas, although mathematics is popularly seen in this way. The guidance certainly constructs such mathematics as unambiguous. To summarise, the NNS guidance implies both a linguistic and a psychological perspective, which derives from the formal model described above. The model of mathematical language is largely concerned with vocabulary, with words co-responding neatly with meanings. Psychologically, the guidance portrays students as acquiring meanings through listening to explanations and having any problems corrected by the teacher. The role of students is largely passive. The teacher instigates a linear process of providing words with their meanings and clarifying where there are problems. Students are then ready to use the new words. The ambiguity practices in this text therefore serve to construct a technicist or bureaucratic view of teaching (see, for example, Alrø & Skovsmose, 2002) in which teachers control the delivery of the mathematics curriculum and the language of that curriculum. The language and curriculum themselves, however, are constructed as givens, supplied by documents such as the vocabulary booklet.

Let me turn, now, to the 'dimensions' transcript extract (introduction, this volume, Appendix 2). What instances of ambiguity arise? How are they dealt with by the participants? What role do they play in developing mathematical ideas?

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After leading a discussion of the nature of two dimensional shapes, the teacher asks:

7 **T:** . . . What's the difference then between two dimensional and three dimensional. W tells us it's **flat** that's fine. Are there anything else to say. F.
8 **F:** Um a (three dimensional shape) has breadth, length and height.
9 **T:** Well done. This would be a two dimensional shape (*draws a square*) (. . .) and a three dimensional shape will have an extra dimension. That would be a **solid** shape (*draws a cube*) okay G.

In this exchange, the teacher invokes a distinction between two and three dimensions and invites the class to describe that distinction. F's offer is accepted by the teacher as implicitly contrasting with earlier formulations of two dimensions as concerning breadth and length, but not height. The teacher also introduces two further distinctions, one visual in the form of two images drawn on the board, and one verbal in the form of two words which are treated as contrasting, 'flat' and solid'. The teacher's contrastive rhetoric could be seen as an example of 'sorting out ambiguities', offering three sets of contrasting pairs each characterising two dimensions versus three dimensions. One of these contrasts is returned to several turns later by student J (turn 40), apparently as part of an exchange about the nature of circles of different dimensions:

36 **V:** And a sphere is three dimensional
37 **T:** And a sphere is three dimensional. What would be a one dimensional circle then
38 **A:** (. . .) a line (*shrugs*)
39 **T:** Just a diameter (*points to diameter from before*). Yes J
40 **J:** (m a two dimensional is flatter . . .)
41 **T:** Yep flat. Look. (*picks up a plastic circle from a set*) I don't like these (. . .) coz they look like three dimensional don't they. They're thick but they're not meant to be, they're meant to be two dimensional. Okay, they're flat shapes (*picks up a square*)

Student J (turn 40) describes a two dimensional shape using the property of flatness first introduced by the teacher (turn 7). The teacher affirms this usage by applying it herself 'yep flat' and then indicates that she is to elaborate, 'Look'. She picks up a circle from a set of brightly coloured plastic shapes. She then makes a kind of disclaimer regarding the shapes, prefacing anything which may follow by saying:

41 **T:** I don't like these (. . .) coz they look like three dimensional don't they. They're thick but they're not meant to be, they're meant to be two dimensional. Okay, they're flat shapes (*picks up a square*)

The disclaimer is marked as being a case of her personal preference, placing what follows in contrast to more 'official' descriptions. Interestingly, from the NNS point of view, the teacher's critique of the plastic shapes appears to be introduce a degree of ambiguity into the discussion: the shapes can be seen as both two dimensional and three dimensional. Hence the teacher's actions are indeed in opposition to the 'official' model of 'explaining meaning clearly' and 'sorting out ambiguities'. Ambiguity, however, is not necessarily the problematic presence of two meanings or interpretations. What do the students and the teacher do with the teacher's introduction of two ways of describing the shapes?

41 **T:** I don't like these (. . .) coz they look like three dimensional don't they. They're thick but they're not meant to be, they're meant to be two dimensional. Okay, they're flat shapes (*picks up a square*)
42 **?** A cylinder
43 **T:** Yeah that's a cylinder (*laughs, waves circle*) (and that's a)
44 **?** a cuboid
45 **T:** cuboid (*waves square*). But it's not meant to be it's meant to be flat. Yes K.
46 **K:** There's no such thing as a one dimensional shape coz a line is kind of like a rectangle filled in
47 **T:** Yeah. What just a line? (*points to board*)
48 **K:** Yeah
49 **T:** Like a- what like [(. . .) (*gestures thinness*)
50 **K:** [a rectangle filled in
51 **T:** (*Giggles*) Very clever. Like a dot (*draws dot*) oops (*erases, does again*) like that. It's interesting isn't it. Yes H?

Some of the students in the class engage with the two modes of description the teacher has proposed, labelling the shapes she is holding as 'cylinder' and 'cuboid', thus drawing on the three dimensional perspective. These labels elicit laughter from the teacher in a manner redolent of someone appreciating a pun (ambiguity often appears to lead to laughter). The laughter serves to mark the label as one possibly deviant choice, a sign of complicity between teacher and students in using the description that isn't 'meant'. The students thus identify with what the teacher has portrayed as her preference. The authorised version is, however, re-emphasised once the joke has been shared: 'it's meant to be flat'.

The exchange in which the teacher shares an alternative perspective and some of the class identify with that position is followed by a statement from student K:

46 **K:** There's no such thing as a one dimensional shape coz a line is kind of like a rectangle filled in

Until K's statement, a clear distinction has been maintained between two possible modes of description, that which is 'meant', versus that which 'looks like' an alternative. K, however, merges the two modes, describing a one dimensional shape as a line, and a line as 'a rectangle [two dimensional] filled in'. Thus a one dimensional shape must be two dimensional. K appears to be arguing that what is 'meant' (in this case a one dimensional shape) is not possible. From the NNS point of view, K's statement may be seen as a sign of confusion, the disastrous consequence of the teacher's unnecessary muddying of the waters. From a discursive perspective, I contend, K's observation takes on a different light. The student has identified a crucial aspect of mathematical discursive practice, namely that what is 'meant' is rarely the same as what things 'look like'. I defined mathematical discursive practices as 'ways of using linguistic resources to do mathematics'. In the case of the use of diagrams, for example, there are a range of referential practices (including pointing, labelling, use of deixis, etc.) which construct diagrams as exemplars which stand for idealised classes of objects. Diagrams of squares, for example, are rarely square: rulers or printing presses are inevitably inaccurate to some degree. In the discourse of mathematics, however, such 'inaccuracies' are not important: a picture of a square is treated as if it is perfectly square for the purposes of mathematical discussion, in the same way that plastic shapes are to be treated as two dimensional when they can also be seen as three dimensional. K's articulation of this idea in the particular case of the representation of one dimensional shapes is an example of mathematical thinking, in the sense that he generalises ideas from the preceding discussion to a new set of situations (any line), accounting for his claim in analogous, though reformulated terms. This accounting continues over five more turns in an exchange with the teacher which draws on gesture and visual images.

K's statement about one dimensional shapes (line 46), and the ensuing discussion, can be seen as an exploration of what it is possible to say using the word dimension. In short, the students are probing and developing aspects of mathematical discourse. In so doing, the use of the term dimension becomes more complex, encompassing a range of mathematical discursive practices. Among these are that any object can be described as having additional dimensions (*a line can be called one dimensional, two dimensional, n-dimensional*); and consequently, that the language of dimension offers different ways of describing mathematical objects, each of which is equally applicable. A plastic shape can be described as two dimensional, and so as square, as having length and breadth, and as being flat. The same shape can also be described as three dimensional, and so as cuboid, having length breadth and depth, as being solid. These descriptions are not mutually exclusive.

The class's discussion explores some relatively sophisticated mathematical ideas. The discussion originates in the teacher's introduction of ambiguity. This ambiguity becomes a *resource* for the participants, which they use to probe the affordances (see Edwards, 1999) of the term dimension. The teacher did not explain the meaning of dimension 'clearly', a task which is in any case problematic (see Morgan, this volume). Rather, the class engaged in a process of joint meaning-making, trying out the possibilities of words which they have encountered before extending their experience of using these words to think mathematically together. Ambiguity can therefore be seen as a learning opportunity rather than a hindrance.

The above analysis suggests that the interaction captured in the 'dimensions' transcript, does not conform to the approach implied by the NNS guidance, reflecting the discursive model rather than the formal model. In particular, meanings are not seen as fixed or as attaching to specific elements of language, such as words. Rather, language, gestures, images, artefacts and so on, are used as resources for thinking. Meanings, rather than being absolute, are relative, flexible and thus amenable to development, deepening and increasing complexity. It is this flexibility that makes possible the development of participants' mathematical ideas, practices and language and the shared meanings of the class. This observation implies in turn that explorations of ambiguity are important opportunities for participants to extend their understanding of both language and mathematics.

Conclusions

The above analysis has highlighted some limitations in the NNS perspective on the use of language in the mathematics classroom. In particular, that perspective appears to see ambiguity as a problem, as something to be avoided and 'sorted out'. It implies a linear model of introducing new words, explaining and clarifying their meanings, with the new words then ready for use. This model does not correspond to the nature of the interaction in the transcript I have discussed. By taking a social, discursive position, ambiguity can be seen as a resource for participants. In the 'dimensions' transcript, ambiguity is used by the teacher and her students to explore how shapes are represented and described, particularly in dimensional terms. They also use ambiguity to identify with each other through sharing humour and complicity in what is portrayed as an unofficial mode of description. K's sophisticated statement regarding the impossibility of drawing a one dimensional shape emerges from these preceding explorations. If ambiguity had been stamped out at every turn, it seems unlikely that the discussion could have reached such a position. This discursive perspective implies a cyclical model, in which new vocabulary is repeatedly encountered and explored in rich, meaningful contexts. Each encounter adds to participants' experience of a part of mathematical discourse and of mathematical thinking, both of which therefore become more complex over time.

More generally, my analysis supports the view (e.g. Moschkovich, 2003) that a rigid distinction between formal and informal language in the mathematics classroom is not necessarily productive. Informal language can be used to explore and develop sophisticated mathematical ideas and to participate in mathematical practices. I do not wish to suggest that students should not learn to use formal aspects of mathematical discourse. This learning, however, is not identical with learning mathematical vocabulary. Rather, students' development of the use of mathematical discourse is intertwined with their development of mathematical thinking. Ambiguity acts as an important resource for students and teachers, serving as a means of articulating between thinking and discourse.

Correspondence

Any correspondence should be directed to Dr Richard Barwell, Graduate School of Education, University of Bristol, 35 Berkeley Square, Bristol BS8 1JA, UK (richard.barwell@bristol.ac.uk).

Note

1. For details of the texts referred to in this paper, which is one of a set, see the introductory paper, this volume, pp. 97–102 'Language in the Mathematics Classroom').

Acknowledgements

I am grateful to the students and teacher who feature in this transcript. This work was supported by ESRC R00429934027.

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