If none of the answers is correct choose e) NOTA. NO calculators.

1. What is the area of a 30° sector of a circle whose circumference is 6π ?

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) π d) $\frac{3\pi}{2}$ e) NOTA

2. A saltshaker is sitting 6 centimeters from the center of a circular lazy Susan. If Ann turned the lazy Susan 120° in $\frac{1}{2}$ second, what is the shaker's linear velocity in mm/sec ?

a) $\frac{2\pi}{3}$ b) $\frac{20\pi}{3}$ c) 8π d) 80π e) NOTA

3. Which of the following is an equation of the line perpendicular to $2x - 3y = 5$, passing through the center of the circle described by $x^2 + y^2 + 2x + 8y + 8 = 0$.

a) $3x + 2y = 11$ b) $2x - 3y = -14$ c) $3x + 2y = -11$ d) $3x + 2y = 5$ e) NOTA

4. Evaluate $\sin(\frac{2\pi}{3}) \bullet \frac{1}{2}\log_3(81) \bullet 10^{\log 3+\log 6-\log 2} \bullet \text{Tan}^{-1}(-1)$.

a) $\frac{-9\pi\sqrt{3}}{4}$ b) $\frac{-27}{4}$ c) $\frac{-18\pi\sqrt{3}}{4}$ d) $\frac{-27\pi\sqrt{3}}{4}$ e) NOTA

5. Which of the following statements is false?

a) $2\sin^2(150°) = 1 - \cos(300°)$ b) $\tan(\frac{\pi}{2} - x) = \cot(x)$

c) $\sin(90° + x) = \cos(x)$ d) $\cos(\frac{2\pi}{3}) = \cos^2(\frac{\pi}{3}) - \sin^2(\frac{\pi}{3})$ e) NOTA

6. In how many ways can two admirals, three generals and four diplomats be seated in a row if the two admirals cannot sit side by side?

a) $9! - 8!$ b) $9! - 8! \bullet 2!$ c) $9! - \binom{9}{2}$ d) $9! - 3 \bullet 2!$ e) NOTA

7. $f(x) = x(1 - x)$ $\frac{f(x+h)-f(x)}{h} =$

a) $1 - 2x$ b) $1 - 2x + h$ c) $-2xh + h - h^2$ d) $1 - 2x - h$ e) NOTA

8. A mural is to be painted on a rectangular museum wall that is 40 ft by 50 ft. A mosaic trim is placed in a uniform strip of a single row of colored square tiles around the inner edge of the rectangle to create a border. 60% of the wall is left for the mural. What is the length in feet of the diagonal of each of the tiles?

a) $2\sqrt{2}$ b) $5\sqrt{2}$ c) 5 d) $10\sqrt{2}$ e) NOTA

9. $f(\Theta) = 2 + 5\cos(4(\Theta + 8^\circ))$. If $0^\circ \le \Theta \le 90^\circ$ for what value of Θ does f reach its minimum value?

a) 14.5° b) 37° c) 82° d) 172° e) NOTA

10. The sum of three consecutive terms of an arithmetic sequence is 30 and their product is 360. What is the smallest of the three numbers?

a) 2 b) 5 c) 10 d) 18 e) NOTA

11. It is given that $\sin(x) = \frac{1}{7}$ and $\tan(x) < 0$. $\sin(2x) =$

a) $\frac{-2\sqrt{13}}{91}$ b) $\frac{4\sqrt{3}}{49}$ c) $\frac{-8\sqrt{3}}{49}$ d) $\frac{-3\sqrt{13}}{91}$ e) NOTA

12. $\frac{2i(5-i)}{2+i} =$

a) $\frac{6}{5} - \frac{18i}{5}$ b) $\frac{14}{3} + \frac{8i}{3}$ c) $\frac{14}{5} + \frac{18i}{5}$ d) $\frac{14}{3} - \frac{18i}{3}$ e) NOTA

13. What is the domain of $f(x) = \ln\left(\frac{x-1}{x+3}\right)$?

a) $(0, \infty)$ b) $(1, \infty)$ c) $(-3, 1)$ d) $(-\infty, -3) \cup (1, \infty)$ e) NOTA

14. The vertices of a triangle coincide with the vertex and the zeroes of $f(x) = x^2 + 2x - 8$. What is the area of the triangle?

a) $2\sqrt{2}$ b) $16\sqrt{2}$ c) 27 d) 54 e) NOTA

15. When a pole casts a shadow of 30 feet, the angle of elevation is Θ. When the same pole casts a shadow of 14 feet, the angle of elevations is 2Θ. Determine the height of the pole to the nearest foot.

a) 7 b) 8 c) 9 d) 12 e) NOTA

16. $f(x) = \frac{(x-a)(x-b)}{3(x-c)^2}, a \ne b \ne c$. Which of the following are the equations of its asymptotes?

a) $x = a, x = b, x = c$ b) $x = a, x = b, y = \frac{1}{3}$ c) $x = c, y = 0$ d) $x = c, y = \frac{1}{3}$ e) NOTA

17. Find the eccentricity of the conic defined by $16x^2 + 25y^2 - 32x + 100y - 284 = 0$.

a) $\frac{3}{5}$ b) $\frac{4}{5}$ c) $\frac{\sqrt{15}}{5}$ d) $\frac{2\sqrt{13}}{15}$ e) NOTA

18. Two forces of magnitudes 10 lb and 12 lb make an angle of $\frac{\pi}{3}$ with each other and are applied to an object at the same point. Find the magnitude of the resultant force.

a) $2\sqrt{3}$ b) $15\sqrt{17}$ c) $2\sqrt{91}$ d) 22 e) NOTA

19. **Find the abscissas of the points having an ordinate of 4 that are a distance of $\sqrt{117}$ from the point (5,-2).**

a) -4 and 14 b) $5 \pm \sqrt{13}$ c) $5 \pm 3\sqrt{17}$ d) -2 and 9 e) NOTA

20. $f(x) = e^{\sqrt{x-2}} + 1$. For $f^{-1}(x) > 2$, $f^{-1}(x) =$

a) $\ln(x-1)^2 + 2$ b) $\ln(x+1)^2 + 2$ c) $\ln^2(x-2) + 1$ d) $[\ln(x-1)]^2 + 2$ e) NOTA

21. $f(x) = \begin{cases} \sqrt{x-1}, x \geq 1 \\ -2x+2, x < 1 \end{cases}$. Which of the following statements is true about f?

a) $\lim_{x \to 1} f(x)$ does not exist. b) f always increases. c) f is continuous at x = 1

d) f has an inverse function e) NOTA

22. Find the area of the figure determined by $y = \frac{\sqrt{4-x^2}}{2}$ and $y = -\sqrt{4-x^2}$.

a) $\frac{5\pi}{2}$ b) 3π c) 4π d) 5π e) NOTA

23. Express $2 \text{ cis}\left(\frac{\pi}{7}\right) \cdot 6 \text{ cis}\left(\frac{29\pi}{42}\right)$ in rectangular form. $r \text{ cis}(\theta) = r(\cos(\theta) + i \sin(\theta))$.

a) $-6\sqrt{3} + 6i$ b) $12 - 12\sqrt{3}\, i$ c) $-12\sqrt{3} + 12i$ d) $-6 + 6\sqrt{3}\, i$ e) NOTA

24. In ΔABC AB = 7, BC = 13 and AC = 15. tan(A) =

a) $\frac{\sqrt{5}}{5}$ b) $\frac{1}{2}$ c) $\sqrt{3}$ d) $\sqrt{5}$ e) NOTA

25. The polar axis and its extension are along the principal axis of a hyperbola having a focus at the pole. The corresponding directrix is to the left of the focus. If the hyperbola contains the point $\left(1,\frac{2\pi}{3}\right)$ and the eccentricity is 2, what is the center of the hyperbola?

a) $\left(\frac{2}{3},\pi\right)$ b) $\left(\frac{4}{3},\pi\right)$ c) (-2,0) d) $\left(0,\frac{2\pi}{3}\right)$ e) NOTA

26. f(x) = $\ln(3\sin(x - \frac{\pi}{4}))$. What is the domain and range of f? For choices a,b,c and d, k is a an integer.

a) $\frac{\pi}{4}+2\pi k < x < \frac{5\pi}{4}+2\pi k, y \le \ln(3)$ b) $\frac{\pi}{4} < x < \frac{5\pi}{4}, 0 \le y \le \ln(3)$

c) $2\pi k < x < \pi + 2\pi k, 0 \le y \le \ln(3)$ d) $\frac{\pi}{4}+2\pi k < x < \frac{5\pi}{4}+2\pi k, 0 \le y \le \ln(3)$ e) NOTA

27. Find the length of the hypotenuse of the right triangle of maximum area whose sum of the lengths of its legs is 5.

a) 2.5 b) 3.125 c) $\frac{5\sqrt{2}}{2}$ d) 12.5 e) NOTA

28. If the average of a,b,c and d is p, and the average of e,f and g is q, what is the average of a,b,c,d,e,f and g in terms of p and q?

a) p + q b) $\frac{p+q}{2}$ c) $\frac{3p+4q}{7}$ d) $\frac{4p+3q}{7}$ e) NOTA

29. P(x) = $x^4 + ax^3 + bx^2 + cx + 24$. P(3) = 0 and P(2i) = 0. Find $a \bullet b \bullet c$, if all three are real numbers.

a) -15 b) 250 c) 1000 d) 24,000 e) NOTA

30. Which of the following statements are true if none of the denominators equal 0?

i) $|\sec(x)| \ge 1$ ii) $2\sum_{b=2}^{8} b^3 + \sum_{b=1}^{8}\frac{1}{b} = \sum_{b=1}^{8}[2(b+1)^3 + \frac{1}{b}]$ iii) $\sqrt[5]{1024a^{-5}b^{15}} = \frac{4b^3}{a}$

ERRATA

ALL ANSWERS Florida Invitational MIDDLETON TIGERS Feb 24, 2007

NO CALCULATORS!

	Algebra I	Geometry	Algebra II	Pre-Calc	Calculus	Statistics	Theta Open	Alpha Open	Statistics Open
	none	1	none	none	1	none	none	none	2
1	C	A	C	B	D	B	B	A	C
2	B	C	B	D	A	C	C	B	D
3	A	C	A	C	A	A	C	A	C
4	D	B	D	A	B	D	D	D	B
5	D	D	C	E	C	B	B	C	A
6	B	D	A	B	~~B~~ E	C	D	C	D
7	B	D	B	D	D	D	A	C	C
8	C	A	B	B	B	C	C	A	~~C~~ → B
9	C	A	C	B	C	C	D	D	A
10	C	C	C	A	A	A	C	B	D
11	A	B	C	C	D	C	B	A	A
12	D	C	D	C	C	B	B	B	B
13	A	A	A	D	D	B	B	B	A
14	B	A	B	C	B	A	A	C	C
15	E	C	C	B	A	D	A	B	D
16	B	D	A	D	C	C	B	A	D
17	C	B	C	A	A	B	B	C	B
18	C	C	C	C	A	D	D	D	A
19	B	B	C	A	D	D	A	D	A
20	B	B	B	D	D	B	D	B	C
21	D	C	D	C	A	B	B	B	D
22	C	D	C	B	C	A	D	C	C
23	C	C	B	A	C	A	A	D	B OR E
24	A	D	C	C	A	B	D	D	D
25	D	B	C	B	D	A	B	C	C
26	B	~~C~~ D	D	A	D	C	C	C	~~A~~ → B
27	B	A	E	C	D	D	D	C	C
28	D	D	D	D	B	D	B	B	C
29	C	C	C	C	A	C	B	A	C
30	C	B	B	D	B	A	D	C	C

Precalculus Individual Test SOLUTIONS

Florida Invitational at MIDDLETON *February 24, 2007*

no calculator allowed

B 1. $C=6\pi$, $r=3$, $A=9\pi$

$\frac{30^O}{360^O}=\frac{1}{12}$ $\frac{9\pi}{12}=\frac{3\pi}{4}$

D 2. $C=12$ Travels 4π cm in $\frac{1}{2}$ sec.

$\frac{4\pi cm}{.5\sec}*\frac{10mm}{1cm}=\frac{80\pi mm}{\sec}$

C 3. $x^2+2x+1+y^2+8y+16=-8+1+16$

$(x+1)^2+(y+4)^2=9$ $C=(-1,-4)$, $m=\frac{-3}{2}$

$3x+2y=11$

A 4. $\frac{\sqrt{3}}{2}\bullet 2\bullet 9\bullet\frac{-\pi}{4}=\frac{-9\pi\sqrt{3}}{4}$

E 5. All are true.

B 6. Nine can sit in a row in 9! ways. The two administrators can sit together is 8! positions in 2 different ways. $9!-8!\bullet 2!$

D 7. $f(x)=x-x^2$ $\frac{(x+h)-(x-h)^2-(x-x^2)}{h}$

$\frac{x+h-x^2-2xh-h^2-x+x^2}{h}=\frac{h-2xh-h^2}{h}=$

$1-2x-h$

B 8. Area of mural$=(50-2x)(40-2x)=.6(50)(40)$

$2000-100x-80x+4x^2=1200$

$4x^2-180x+800=0$, $x^2-45x+200=0$

$(x-40)(x-5)=0$, $x=5$

B 9. $\frac{360^O}{4}=90^O$ is the period. It reaches its minimum at half the period. $-8^O+45^O=37^O$

A 10. $a+a+d+a+2d=30$, $3a+3d=30$, $a+d=10$

$a(a+d)(a+2d)=360$, $a(10)(a+2d)=360$

$a(a+2d)=36$, $a^2+2ad=36$, $a^2+2a(10-a)=36$

$a^2+20a-2a^2=36$, $a^2-20a+36=0$

$(a-2)(a-18)=0$, $a=2$

C 11. $\sin(x)=\frac{1}{7}$, x is in quadrant 2

$\sin(2x)=2(\frac{1}{7})(\frac{-4\sqrt{3}}{7})=\frac{-8\sqrt{3}}{49}$

C 12. $\frac{10i-2i^2}{2+i}=\frac{2+10i}{2+i}\bullet\frac{2-i}{2-i}=$

$\frac{4-2i+20i-10i^2}{5}=\frac{14+18i}{5}$

D 13. $\frac{x-1}{x+3}>0$

C 14. $x^2+2x-8=0$, $(x+4)(x-2)=0$ Base is length 6. Vertex occurs at -1. $(-1)^2-2-8=-9$

$.5(6)(9)=27$

B 15. $\tan\theta=\frac{h}{30}$, $\tan 2\theta=\frac{h}{14}$,

$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$, $\frac{h}{14}=\frac{2\left(\frac{h}{30}\right)}{1-\frac{h^2}{900}}$

$\frac{h}{14}=\frac{60h}{900-h^2}$

$900h-h^3=0$

$h(60h-h^2)=0$

$h=\sqrt{60}$ therefore 8

D 16. $x=c$ vertical, $y=\frac{1}{3}$ horizontal

A 17. $16(x^2-2y+1)+25(y^2+4y+4)=284+16+100$

$16(x-1)^2+25(y+2)^2=400$

$\frac{(x-1)^2}{25}+\frac{(y+2)^2}{16}=1$

$c^2=25-16$

$c=3$

$\frac{c}{a}=\frac{3}{5}$

C 18. $\sqrt{1}$ = <10, 0> , $\sqrt{2}$ = <6, 6$\sqrt{3}$ >

$\sqrt{256+108}$ = $\sqrt{364}$ = $2\sqrt{91}$

A 19. $\sqrt{(x-5)^2+(-2-4)^2}=\sqrt{117}$

$(x-5)^2+36=117$

$(x-5)^2=81$

x = -4 , x = 14

D 20. x = $e^{\sqrt{y-2}}+1$

x - 1 = $e^{\sqrt{y-2}}$

ln(x - 1) = $\sqrt{y-2}$

$\ln^2$ (x - 1) = y - 2

$\ln^2$ (x - 1) + 2 = y

C 21.

B 22. 2y = $\sqrt{4-x^2}$ $\quad \frac{1}{2}\pi ab=\frac{1}{2}\pi(2)(1)=\pi$

$4y^2 = 4 - x^2$ $\quad \frac{1}{2}\pi r^2=2\pi$

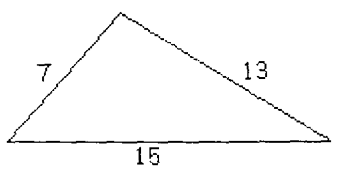
$x^2 - 4y^2 = 4$ $\quad 3\pi$

$\frac{x^2}{4}+\frac{y^2}{1}=1$

A 23. 12cos$\frac{35\pi}{42}$ = 12cos $\frac{5\pi}{6}$ = 12 $\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$ =

$-6\sqrt{3}+6i$

C 24.



169 = 49 + 225 - 2(7)(15) cosA

169 = 274 - 210 cosA

-105 = -210 cosA $\quad$ cosA = $\frac{1}{2}$

A = 60^0 therefore $\sqrt{3}$

B 25. $r=\frac{2d}{1-2\cos\theta}$, $1=\frac{2d}{1-2(-.5)}$, d=1

$r=\frac{2}{1-2\cos\theta}$

The vertices occur at θ=0, $\theta=\pi$

Therefore (-2,0) and ($\frac{2}{3}$, π)

The center is on the principal axis halfway between the two vertices. Therefore, $(\frac{4\pi}{3},\pi)$

A 26. 3sin(y-$\frac{\pi}{4}$) is 3, therefore y ≤ ln3

3sin(y--$\frac{\pi}{4}$) > 0

C 27. x+y=5, A= $\frac{1}{2}$xy. A(y)= $\frac{1}{2}$y(5-y)

0= $\frac{1}{2}$y(5-y) $\qquad$ A(y) = $\frac{5}{2}y-\frac{y^2}{2}$

Vertex at y=$\frac{5}{2}$, therefore $\frac{5\sqrt{2}}{2}$

D 28. $\frac{a+b+c+d}{4}=p \quad \frac{e+f+g}{3}=q$

$\frac{a+b+c+d+e+f+g}{7}=\frac{4p+3q}{7}$

C 29. (2i)(-2i)(3)d=24, 12d=24, d=2

2i-2i+3+2=-a, a=-5

(2i)(-2i)+(2i)(3)+(-2i)(3)+(-2i)(2)+(2i)(2) +2(3) = b, 4+6=b, b=10

(2i)(-2i)(3) + (2i)(-2i)(2) + (-2i)(3)(3) + (2i)(3)(2) = -c, 12+8=-c, c=-20

$abc=1000$

D 30. I) T

II) F first part of right hand of equation would be equal if b went from 1 to 7.

III) T

IV) T

V) T

Pre-Calc Team Question #1 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. $f(x) = x^4 - x^3 - 13x^2 + x + 12$. Let A be the remainder when f is divided by x – 3.

Let B be the **sum** of the complete list of possible positive rational roots as determined by the Rational Root Theorem.

Let C be the product of the roots of f.

Let D be $f(2)$ divided by the greatest number of possible positive real roots determined by Descartes Rules of Signs.

Pre-Calc Team Question #2 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

2. Solve each of the following equations.
A) Let A be the sum of the solutions to the equation $5|A+7| - 3 = 27$

B) $2^{2B+1} = 32$

C) $\log_2(C-3) + \log_2(C+3) = \log_2(16)$

D) $5(4D)^{\frac{3}{2}} = 135$

Pre-Calc Team Question #3 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

3. Let A be the length of the diameter of the circle determined by $x^2 + y^2 - 26x = 0$.

Let B be the length of the latus rectum or focal chord of $y^2 - 4x + 2y + 5 = 0$.

Let C be the length of the distance from the center to a focus of the ellipse determined by $9x^2 + 4y^2 - 18x + 16y - 11 = 0$.

Let D be the length of the transverse axis of the of the hyperbola determined by $25x^2 - 9y^2 - 100x - 72y - 269 = 0$.

Pre-Calc Team Question #4 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

4. Two regular fair dice are thrown. **If the two numbers shown are different**, find the probabilities of each of the following.

A) Their sum is 5.

B) One die shows a 1.

C) Their sum is greater than 9.

D) Their sum is 4 or less.

Pre-Calc Team Question #5 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

5. $(2+2i)^4 = A$

$\mathrm{Tan}^{-1}(\frac{3}{4}) + \mathrm{Tan}^{-1}(\frac{5}{12}) = \mathrm{Tan}^{-1}(\frac{x}{y})$, where x and y are relatively prime. Let B = x + y.

Let C be the sum of the coordinates of the center of the ellipse determined by $x^2 + 4y^2 + 6x - 8y + 9 = 0$.

Let D be the period of $y = 3\sin(\frac{\pi x}{4} + 3)$.

Pre-Calc Team Question #6 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

6. Simplify each of the following to a single trigonometric term or a numerical value, where each expression is defined.

A) $-\cos(\frac{\pi}{2} + x)$

B) $\sin(2x)(\cot(x) + \tan(x))$

C) $\frac{\sec(x)-1}{\sec(x)+1} + \frac{\cos(x)-1}{\cos(x)+1}$

D) $\frac{1+\tan^2(x)}{\tan(x)\csc^2(x)}$

Pre-Calc Team Question #7 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

7. Let A = $(\sqrt{6+3\sqrt{3}} - \sqrt{6-3\sqrt{3}})^2$.

Find B where $\log_B \sqrt{3} = \frac{1}{6}$.

Find C where $\log_3(\log_2(\log_C 256)) = 1$.

Let D = $(\sqrt{\sin(\frac{\pi}{6})})^{-6}$.

Pre-Calc Team Question #8 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

8. In a physics experiment with a weighted spring system, the motions of the weight can be described by the equation $y = 2\sin(\pi t - \frac{\pi}{2})$, where y is the distance in inches from the equilibrium point and t, $t \geq 0$, is the time in seconds.

Let A be the initial distance of the weight from the equilibrium point?

Let B be the distance of the weight from the equilibrium point of the weight after 3 seconds.

Let C be the first time in seconds, that the weight is at the equilibrium point.

Let D be the first time in seconds that the weight is the maximum positive directed distance from the equilibrium point.

Pre-Calc Team Question #9 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

9. If $M = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$, then $M^{-1} = \begin{bmatrix} x & y \\ A & z \end{bmatrix}$. Find A.

If $f(x) = x^3 + 1$, let $B = f^{-1}(28)$.

Let C be the coefficient of the third term in the expansion of $(3a - 2b)^4$.

Let D be the number of ways that five different algebra books and four different geometry books can be arranged if all the algebra books must be together.

Pre-Calc Team Question #10 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

10. A sector of a circle has a perimeter of 12 cm and an area of 8 cm^2.
There are two possible values for the radius of the circle in cm.

Let A be the smaller value and B be the larger value.

The area of ΔMHS is 90 cm^2, m = 18 cm, and h = 15 cm.

Let C be the value of Sin (S).

A sector of a circle of radius 6 has a central angle measure of 30°.
Let D be the perimeter of the sector.

Pre-Calc Team Question #11 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

11. Let A be the **slope** of the rectangular form of the polar equation $1 = r\cos(\Theta + 30^\circ)$.

Let B be the **x-intercept** of the parametric equation x = 3 + 2t and y = -1 + 5t.

Let C be the **y-intercept** of the asymptote of negative slope of the hyperbola described by $25y^2 - 9x^2 - 100y - 72x - 269 = 0$.

Let D be the y-intercept of the line of negative slope passing through the point (2,5) that makes an angle of radian measure $\frac{\pi}{4}$ with the line $3x - 4y = -8$.

Pre-Calc Team Question #12 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

12. The area of ΔABC is 24cm^2, a = 6 cm, b = 10 cm. Angle C is acute.

Let A be the value of sin(C).

Let B be the length of side c in cm.

An equilateral triangle with sides of length 4 cm is drawn within ΔABC.
Let C be the probability that a dart landing in ΔABC lands within the equilateral triangle.

An isosceles triangle is formed by moving 4 units from C to A and 4 units from C to B.
Let D be the ratio of the area of this triangle to ΔABC.

Pre-Calc Team Question #13 Florida Invitational Middleton TIGERS February 24, 2007

NO CALCULATOR

13. Let A be the third term in an arithmetic sequence that has a first term of 7 and has 7 arithmetic means between 7 and -2.

$B = \sum_{i=0}^{\infty} 6\left(\frac{2}{3}\right)^{i}$

Let C be the ratio of the 12^{th} term to the 4^{th} term of the geometric sequence $\sqrt{2}, 2, \cdots$.

Let D be the third term of an arithmetic sequence whose fifteenth term is 38 and whose common difference is -3.

Pre-Calc Team Question #14 Florida Invitational Middleton TIGERS February 24, 2007

NO CALCULATOR

14. Four fair dice are thrown at the same time. Find the probability that

A) Exactly one 4 will show.

B) At least three 4's will show.

C) No 4's will show.

D) All show 4's

15. Solve for matrix X. $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} - \begin{bmatrix} 0 & 6 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 2 & 0 \end{bmatrix}$.

Find the values of A, B, C and D

Pre-Calculus Team Solutions Florida Invitational MIDDLETON TIGERS February 24, 2007
no calculator allowed

1. A. 3| 1 -1 -13 1 12
3 6 -21 -60
1 2 -7 -20 **-48**

B. {1,2,3,4,6,12} = **28**

C. $\frac{12}{1}$= **12**

D. 2| 1 -1 -13 1 12
2 2 -22 -42
1 1 -11 -21 -30

+ - - + + 2 changes, $\frac{-30}{2}$= **-15**

2. 5|A+7|=30, |A+7|=6, A=-1 or A=-13, **-14**

$2^{2B+1} = 2^5$ = 2B+1=5, B=**2**

$\log_2(c^2-9) = \log_2 16$, c^2-9=16, c^2=25, c=**5**

$(4D)^{\frac{3}{2}} = 27$, 4D=9, D=$2\frac{1}{4}$, **2.25**

3. x^2-26x+169+y^2=169, $(x-13)^2+y^2=13^2$, A=**26**

y^2+2y+1=-4x+1-5, $(y+1)^2$=4(x-1), B=**4**

$9x^2-18x+4y^2+16y=11$

$9(x^2-2x+1)+4(y^2+4y+4)=11+9+16$

$9(x-1)^2+4(y+2)^2=36$

$\frac{(x-1)^2}{4}+\frac{(y+2)^2}{9}=1$, c^2=5, c=$\sqrt{5}$

$25(x^2-4x+4)-9(y^2+8y+16)=269+100-144$

$25(x-2)^2-9(y+4)^2=225$

$\frac{(x-2)^2}{9}-\frac{(y+4)^2}{25}=1$, a^2=9, a=3, D=**6**

4. Doubles are eliminated from sample space.

A. (1,4)(4,1)(2,3)(3,2) $\frac{4}{30}=\frac{2}{15}$

B. (1,2)(2,1)(1,3)(3,1)(4,1)(1,4)(1,5)(5,1)
(1,6)(6,1) $\frac{10}{30}=\frac{1}{3}$

C. (4,6)(6,4)(5,6)(6,5) $\frac{4}{30}=\frac{2}{15}$

D. (1,2)(2,1)(1,3)(3,1) $\frac{4}{30}=\frac{2}{15}$

5. A. $[(2+2i)^2]^2 = (4+8i+4i^2)^2 = (8i)^2 =$ **64**

B. $\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}(\frac{5}{12})}$ = $\frac{36+20}{48-15}$ = $\frac{56}{33}$, B=**89**

C. $x^2+6x+9+4(y^2-2y+1)=-9+9+4$

$(x+3)^2+4(y-1)^2=4$, -3+1=-2, C=**-2**

D. $\frac{2\pi}{\frac{\pi}{4}}$=**8**

6. $-[\cos\frac{\pi}{2}\cos x-\sin\frac{\pi}{2}\sin x]$
$=-[0-\cos x-1\cdot\sin x]=\sin x$

7. $6+3\sqrt{3}-2\sqrt{6+3\sqrt{3}}\sqrt{6-3\sqrt{3}}+6-3\sqrt{3}$
$=12-2\sqrt{36-27}=12-2\sqrt{9}=12-6=6$

$B^{\frac{1}{6}}=\sqrt{3}$

$B=\left(\sqrt{3}\right)^6=27$

$\left(\log_2\left(\log_c 256\right)\right)=3$

$\log_c 256=8$

$c^8=256, c=2$

8. A) $2\sin(-\frac{\pi}{2})=-2$, **A=-2**

B) $2\sin(3\pi-\frac{\pi}{2})=2\sin(\frac{5\pi}{2})=2$, **B=2**

C) $2\sin(\pi t-\frac{\pi}{2})=0, \pi t-\frac{\pi}{2}=0, \pi(t-\frac{1}{2})=0$,

$t=\frac{1}{2}$, **C=.5**

D) period $\frac{2\pi}{\pi}=2, t=1$, **D=1**

9. A) $M^{-1}=\frac{1}{10}(-4)=\frac{-2}{5}$, A=$\frac{2}{5}$, **A=.4**

B) $f(3)=28, f^{-1}(28)=3$, **B=3**

C) $6(3a)^2(-2b)^2=6(9a^2)(4b^2)=54\times4=216$,
C=216

D) Algebra books can be arranged in 5! ways. Geometry books in 4! different ways. They can be placed together in 5 different ways.
$5\times120\times24=14400$, **D=14400**

10. $r+r+\frac{\theta}{360°}\times2\pi r=12$,

$\frac{\theta}{360°}\times\pi r^2=8, \frac{\theta}{360°}=\frac{8}{\pi r^2}$

$r+r+\frac{8}{\pi r^2}\times2\pi r=12$, $2r+\frac{16}{r}=12$

$2r^2+16=12r$, $r^2-6r+8=0$, $r=2,4$,

A=2, B=4

C) $90=\frac{1}{2}(18)(15)\sin C$, $90=135\sin C$,

$\frac{90}{135}=\sin C$, $\sin C=\frac{2}{3}$

D) $6+6+\frac{1}{12}(12\pi)=12+\pi$

11. A) 1 = $r(\cos\Theta\cos 30° - \sin\Theta\sin 30°)$

1 = $r\frac{\sqrt{3}}{2}\cos\Theta - r\frac{1}{2}\sin\Theta$

1 = $\frac{\sqrt{3}}{2}x-\frac{1}{2}y$, A = $\sqrt{3}$

B) $y=0$, $0=-1+5t$, $t=\frac{1}{5}$

$x=3+2(\frac{1}{5})=3\frac{2}{5}$, **B=3.4**

C) $25(y^2-4y+4)-9(x^2+8x+16)=269+100-144$

$25(y-2)^2-9(x+4)^2=225,\ \frac{(y-2)^2}{9}-\frac{(x+4)}{25}=1$

m=$\frac{-3}{5}$, 3x+5y=2, c=$\frac{-2}{5}$, **C=.4**

D) $\frac{\frac{3}{4}-m}{1+\frac{3}{4}m}=1,\frac{3-4m}{4+3m}=1, 3-4m=4+3m, -1=7m$

$m=\frac{-1}{7}$, $x+7y=37, D=\frac{37}{7}$

12. A) $24=\frac{1}{2}(6)(10)\sin C,\ 24=30\sin C\ ,\sin C=\frac{4}{5}$

B) $c^2=36+100-2(6)(10)(\frac{3}{5})$,

$c^2=136-72=64$, c=8

C) $\frac{\frac{16\sqrt{3}}{4}}{24}=\frac{\sqrt{3}}{6}$

D) $\frac{1}{2}(4)(4)(\frac{4}{5})(\frac{1}{24})=\frac{4}{15}$

13. A) $-2=7+8d$, $-9=8d$, $d=\frac{-9}{8}$

$a_3=7+2(\frac{-9}{8})=7-\frac{9}{4}=4\frac{3}{4}$

B) $\frac{6}{1-\frac{2}{3}}=\frac{6}{\frac{1}{3}}=18$

C) $\frac{\sqrt{2}(\sqrt{2})^{11}}{\sqrt{2}(\sqrt{2})^{3}}=(\sqrt{2})^8$=16 , 16:1

D) 38= $a_1+(14)(-3)$, $a_1=80$,

$a_3=80+2(-3)=74$

14. A) $\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot 4$

B) $\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5^3}{6}\cdot 4+\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\frac{21}{1296}=\frac{7}{432}$

C) $\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{16}=\frac{625}{1296}$ D) $\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{1296}$

15. $\begin{bmatrix}2&1\\3&2\end{bmatrix}\begin{bmatrix}A&B\\C&D\end{bmatrix}=\begin{bmatrix}1&13\\5&5\end{bmatrix}$,

$Inverse: \frac{1}{7}\begin{bmatrix}2&1\\-3&2\end{bmatrix}$

$\begin{bmatrix}A&B\\C&D\end{bmatrix}=\begin{bmatrix}\frac{2}{7}&\frac{1}{7}\\\frac{-3}{7}&\frac{2}{7}\end{bmatrix}\begin{bmatrix}1&13\\5&5\end{bmatrix}=\begin{bmatrix}1&\frac{31}{7}\\1&\frac{-29}{7}\end{bmatrix}$

A=1, B=$\frac{31}{7}$, C=1, D=$\frac{-29}{7}$