The abbreviation "NOTA" denotes "None of These Answers."
Diagrams are not drawn to scale.

NO CALCULAOTRS ALLOWED

1. $\angle A$ is complementary to $\angle B$. $\angle C$ is complementary to $\angle B$. If $m\angle A = (3x + y)°$, $m\angle B = (x + 4y + 2)°$, and $m\angle C = (3y - 3)°$, find $m\angle B$ in degrees.

   A. 57 B. 33
   C. 123 D. 12 E. NOTA

2. Find the perimeter of a right triangle whose area is 30 and whose hypotenuse is 13.

   A. $23 + \sqrt{69}$ B. $16 + 2\sqrt{34}$
   C. 30 D. $7 + 3\sqrt{13}$ E. NOTA

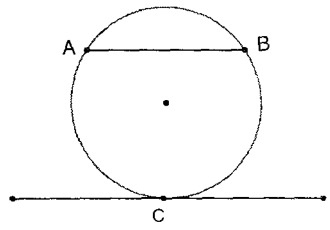
3. A regular pentagon is inscribed in a circle of radius 7. How long is the circular arc that connects two neighboring vertices?

   A. 1.4π B. 2.1π
   C. 2.8π D. 7π E. NOTA

4. Find the altitude of the longest side of a triangle with sides of 12, 10, and 8.

   A. $\frac{5}{4}\sqrt{7}$ B. $\frac{5}{2}\sqrt{7}$
   C. 60 D. 262.5 E. NOTA

5. In the diagram, chord $\overline{AB}$ is parallel to a line tangent to the circle at point C. If the distance between the chord and the tangent line is 18, and the radius of the circle is 13, how long is chord $\overline{AB}$?



   A. 12 B. $2\sqrt{105}$
   C. $2\sqrt{155}$ D. 24 E. NOTA

6. Three circles of radii 2, 4, and 6 are tangent to each other externally. Find the area of the triangle formed by connecting their centers.

   A. 4 B. 6
   C. 12 D. 24 E. NOTA

7. Find the area of the circle that passes through $(9,-4)$ and whose center is $(-3,5)$.

   A. 15π B. 30π
   C. 117π D. 225π E. NOTA

8. Each side of a triangle has length 2x centimeters. Using each vertex as a center, a circle is drawn with a radius of x centimeters. Find the area, in sq cm, bounded by the three arcs that lie within the triangle.

A. $x^2\left(\sqrt{3}-\frac{\pi}{2}\right)$ B. $x^2\left(2\sqrt{3}-\pi\right)$
C. $2x^2\left(2\sqrt{3}-\pi\right)$ D. $\sqrt{3}x^2-\pi$
E. NOTA

9. Each diagonal of a rhombus is 6 units long. Give the area of the rhombus in square units.

A. 18 B. 12
C. 6 D. 36 E. NOTA

10. Each of the degree measures of the equal angles of an isosceles triangle exceeds twice the degree measure of the vertex angle by $15°$. Find the degree measure of the vertex angle.

A. 20 B. 25
C. 30 D. 35 E. NOTA

11. Given: right ΔADC with altitude to the hypotenuse drawn and labeled $\overline{DB}$, with B on the hypotenuse. If AD=6 and BC=5, find the length of $\overline{DB}$.

A. 3 B. $2\sqrt{5}$
C. $3\sqrt{5}$ D. not enough information
E. NOTA

12. The lengths of the three sides of a triangle are given as $8x, 15-2x$ and $x+25$. The interval in which x must lie is written as $A<x<B$. Find the product of A and B.

A. $\frac{20}{11}$ B. 15
C. $\frac{80}{9}$ D. $\frac{100}{3}$ E. NOTA

13. A triangle has two angles with measures of $45°$ and $30°$. The length of the side included by these two angles is 6. Find the area of the triangle.

A. $9\sqrt{3}-9$ B. $12-2\sqrt{3}$
C. $18-3\sqrt{3}$ D. $27-9\sqrt{3}$ E. NOTA

14. Point P divides $\overline{AB}$ into two segments so that $\frac{AP}{PB}=\frac{3}{5}$. Find the length of $\overline{AP}$ if $AB=40.$

A. 15 B. $\frac{200}{3}$
C. 25 D. $\frac{80}{3}$ E. NOTA

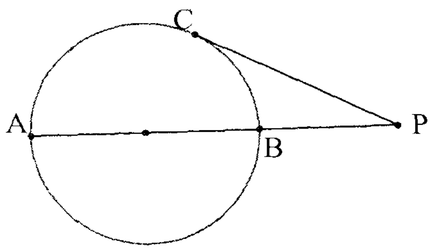
15. The area of a circle is $\frac{3}{4}$ the area of a square. One side of the square has length $2\sqrt{6\pi}$. Find the length of the diameter of the circle.

A. $3\sqrt{2}$ B. $2\sqrt{3}$
C. $6\sqrt{2}$ D. $2\sqrt{6}$ E. NOTA

16. A regular polygon has 10 sides. Find the degree measure of one interior angle.

A. 100 B. 140
C. 134 D. 144 E. NOTA

17. In the diagram, $\overline{AB}$ is a diameter. If tangent segment $CP = 10$ and secant segment $AP = 20$, find the length of the radius of the circle.



A. 6 B. $7\frac{1}{2}$
C. 10 D. 12 E. NOTA

18. $\angle 1 \cong \angle 2$ and $m\angle 3 = (x+18)°$. The $m\angle 1$ is $36°$ more than the measure of the complement of $\angle 3$. The $m\angle 2$ is $9°$ more than half the measure of the supplement of $\angle 3$. Find the average of the degree measures for $\angle 1$ and $\angle 2$.

A. 36 B. 54
C. 72 D. 108 E. NOTA

19. The side lengths of a triangle are in the ratio 3:4:5. The perimeter is $12\sqrt{2}$. Find the area of the triangle.

A. $6\sqrt{2}$ B. 12
C. 20 D. 24 E. NOTA

20. In circle O, from an external point P, tangent segment $\overline{PB}$ is drawn. A secant segment is also drawn from P with endpoint C on the circle and intersecting the circle at point A. If $PB = 6, PC = 12,$ find the length of $\overline{AC}$.

A. 6 B. 9
C. 12 D. 15 E. NOTA

21. The length of a rectangle is 2 feet less than its width. If the length is increased by 4 feet, and the width is decreased by 2 feet, the area of the new rectangle is 8 sq ft greater than the original. Find the area of the original rectangle.

A. 8 B. 12
C. 24 D. 32 E. NOTA

22. ΔABC has D on $\overline{AB}$ and E on $\overline{AC}$ such that $\overline{DE} \parallel \overline{BC}$. If $AD = x-2$, $EC = x+3, BD = 9, DE = 5, AE = 4,$ find the perimeter of ΔABC.

A. 13 B. 29.25
C. 37.25 D. 42.25 E. NOTA

23. In triangle ABC, side $\overline{AB}$ is x longer than side $\overline{BC}$. The ray bisecting angle B intersects side AC at point D. If $AC = 8$ and $CD = 3$, find the perimeter of the triangle in terms of x.

A. $\frac{29}{15}$ B. $\frac{4}{3}x + \frac{8}{3}$
C. $4x+8$ D. $3x+8$ E. NOTA

24. Carol's tennis ball has a radius of 2, and David's basketball has a radius of 20. If the balls are lying tangent to each other and to the floor, what is the distance between the points of tangency on the floor?

A. 18 B. 22
C. $4\sqrt{30}$ D. $4\sqrt{10}$ E. NOTA

25. An isosceles trapezoid has base angle of $60°$ and base lengths of 8 and 14. Find the perimeter of the trapezoid.

A. 28 B. 34
C. 56 D. $36\sqrt{3}$ E. NOTA

26. The ratio of the area of a circle to its circumference is 8. Find the diameter of the circle.

A. 8 B. 12
C. 16 D. 32 E. NOTA

27. To get a rectangular photograph which is 3 inches longer than it is wide to fit into a rectangular frame, Martin had to trim a one-inch strip from each edge of the photo. In all, he trimmed off 46 sq inches. What were the original dimensions in inches?

A. 11 x 14 B. 13 x 10
C. 11 x 8 D. 9 x 12 E. NOTA

28. Isosceles right triangles are cut from the four corners of a square piece of paper, 12 inches by 12 inches, so that a regular octagon is produced. What is the length, in inches, of each leg of these isosceles right triangles?

A. 4 B. $2+\sqrt{2}$
C. $6-\sqrt{2}$ D. $12-6\sqrt{2}$ E. NOTA

29. The length of the diagonal of a square is the perimeter of a right triangle, and the lengths of the shorter two sides of the triangle are 3 and 4, find the area of the square in square units.

A. 12.5 B. 25
C. 72 D. 144 E. NOTA

30. If a circle with a radius of 10 has its radius decreased by 2 , by what percent is the area decreased?

A. 20% B. 36%
C. 40% D. 45% E. NOTA

ERRATA

ALL ANSWERS Florida Invitational MIDDLETON TIGERS Feb 24, 2007

NO CALCULATORS!

	Algebra I	Geometry	Algebra II	Pre-Calc	Calculus	Statistics	Theta Open	Alpha Open	Statistics Open
	none	1	none	none	1	none	none	none	2
1	C	A	C	B	D	B	B	A	C
2	B	C	B	D	A	C	C	B	D
3	A	C	A	C	A	A	C	A	C
4	D	B	D	A	B	D	D	D	B
5	D	D	C	E	C	B	B	C	A
6	B	D	A	B	~~B~~ E	C	D	C	D
7	B	D	B	D	D	D	A	C	C
8	C	A	B	B	B	C	C	A	~~C~~ → B
9	C	A	C	B	C	C	D	D	A
10	C	C	C	A	A	A	C	B	D
11	A	B	C	C	D	C	B	A	A
12	D	C	D	C	C	B	B	B	B
13	A	A	A	D	D	B	B	B	A
14	B	A	B	C	B	A	A	C	C
15	E	C	C	B	A	D	A	B	D
16	B	D	A	D	C	C	B	A	D
17	C	B	C	A	A	B	B	C	B
18	C	C	C	C	A	D	D	D	A
19	B	B	C	A	D	D	A	D	A
20	B	B	B	D	D	B	D	B	C
21	D	C	D	C	A	B	B	B	D
22	C	D	C	B	C	A	D	C	C
23	C	C	B	A	C	A	A	D	B OR E
24	A	D	C	C	A	B	D	D	D
25	D	B	C	B	D	A	B	C	C
26	B	~~C~~ D	D	A	D	C	C	C	~~A~~ → B
27	B	A	E	C	D	D	D	C	C
28	D	D	D	D	B	D	B	B	C
29	C	C	C	C	A	C	B	A	C
30	C	B	B	D	B	A	D	C	C

SOLUTIONS

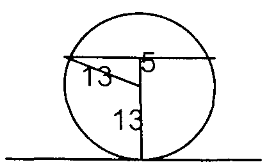
A 1. $3x+y+x+4y+2=90, 4x+5y=88;$
$x+4y+2+3y-3=90, x+7y=91;$
Solving this system gives x=7,y=12.
So $m\angle B = 57$.

C 2. The hypotenuse is 13. One side is x and the other is 17-x. Since the triangle is right, use the area formula to solve and find that x = 12 or 5. Therefore the perimeter is 30.

C 3. One central angle would be 72.
$\frac{72}{360}\cdot 14\pi = 2.8\pi$

B 4. Find the area using Heron's formula. That would be $15\sqrt{7}$. Using the area formula $\frac{1}{2}bh$ gives
$15\sqrt{7}=\frac{1}{2}\bullet 12\bullet h, h=\frac{5\sqrt{7}}{2}$

D 5.



This forms a 5-12-13 right triangle. The chord has length 24.

D 6. Drawing the three circles and their radii, you get the sides of the triangle to be 6, 8, 10. This is a right triangle making the area 24.

D 7. Use the distance formula.
$\sqrt{12^2+9^2}=15$. So the area is 225π.

A 8. The triangle is equilateral. Drawing the three circles gives us a section that is the triangle minus three $60°$ arcs of a circle. So the area of the section would be the area of the triangle minus a semicircle. The area of the triangle is $\frac{4x^2\sqrt{3}}{4}$ which is $x^2\sqrt{3}$ and the semicircle is $\frac{1}{2}\cdot x^2\pi$.
The area of the section is
$x^2\sqrt{3}-\frac{x^2\pi}{2}=x^2\left(\sqrt{3}-\frac{\pi}{2}\right)$.

A 9. The rhombus must be a square since the diagonals are congruent. The area is $\frac{d^2}{2}=18$.

C 10. Let the vertex angle have measure x. The congruent angles will be $2x+15$ so $x+2(2x+15)=180, x=30$ which is the vertex angle.

B 11. Since the triangle is right and we have the altitude to the hypotenuse, we have similar triangles which means we can use the geometric mean theorems.
Let AB=x, then $6=\sqrt{x(x+5)}, x=4$.
To find DB, $\sqrt{4\cdot 5}=2\sqrt{5}$.

C 12. Using the fact that the sum of any two sides must be greater than the third, we must do 3 inequalities since we don't know which side is the largest. (Continued)

#12 con't

$8x+15-2x>x+25$ gives $x>2$.

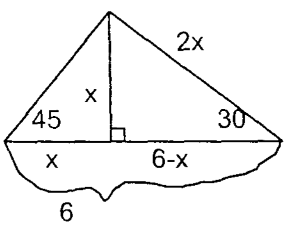
$-x+40>8x$ gives $x<\frac{40}{9}$.

$9x+25>15-2x$ gives $x>-\frac{10}{11}$ so reject this one.

In addition, each side must be positive so $15-2x>0, x<\frac{15}{2}$. The inequality we have is now $2<x<\frac{40}{9}$.

The product of those is $\frac{80}{9}$.

A 13. Using the diagram below, and measures as given and altitude drawn. The side with measure 6-x must also be $x\sqrt{3}$ since the triangle is 30-60-90. So, $6-x=x\sqrt{3}$. Solving this gives $x=-3+3\sqrt{3}$. The area would be $\frac{1}{2}\cdot 6\cdot\left(-3+3\sqrt{3}\right)=-9+9\sqrt{3}$.

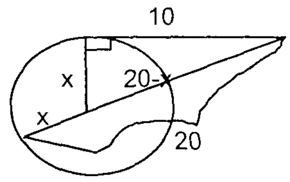


A 14. Let A ... gives the equat ... he length of segment AP is 15.

C 15. $\pi r^2=\frac{3}{4}s^2$. Solving gives $r=3\sqrt{2}$ so diameter would be $6\sqrt{2}$.

D 16. One exterior angle has a measure of 36 so the interior angle is the supplement of this which is 144.

B 17. Using the diagram below and the Pythagorean Theorem gives the equation $x^2+100=400-40x+x^2$. $x=\frac{15}{2}$ which is the radius.



C 18. $m\angle 1=36+(90-x-18)$,

$m\angle 2=9+\frac{1}{2}(180-x-18)$,

Since these two angles are congruent, set them equal to each other and solving to get $x=36. \angle 1=72$ and so does $\angle 2$ so the average is 72.

B 19. $12x=12\sqrt{2}$, sides area $3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}$, a right triangle. So the area is $\frac{1}{2}\bullet 3\sqrt{2}\bullet 4\sqrt{2}$ which is 12.

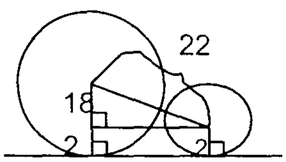
B 20. The length of the tangent squared equals the length PA times the length PC. $36=12x, x=3$, making AC =9.

C 21. The dimensions of the original rectangle are w and $w-2$ making the area w^2-2w. The new rectangle has dimensions $w-2$ and $w+2$ making its area w^2-4. The equation is now $w^2-4=8+w^2-2w, w=6$. The actual dimensions of the original rectangle would now be 6 and 4 which gives an area of 24.

D 22. Since the lines are parallel, the two triangle are similar by AA so the corresponding sides are proportional. Solving for x, $\frac{x-2}{9}=\frac{4}{x+3}, x=6$. This makes AD=4 and AB=13. Solving for y, $\frac{5}{y}=\frac{4}{13}, y=\frac{65}{4}$. The perimeter is 13+13+16.25 which is 42.25.

C 23. Let side AB = x+y. Side BC = y. Using the triangle angle bisector theorem, $\frac{x+y}{5}=\frac{y}{3}, y=\frac{3}{2}x$. Side BC = $\frac{3}{2}x$, AB=x+y=$\frac{5}{2}x$,AC=8. Perimeter is 4x+8.

D 24. Using the diagram below and Pythagorean Theorem, $\sqrt{22^2-18^2}=\sqrt{160}=4\sqrt{10}$.



B 25. Drawing the altitudes gives two congruent 30-60-90 triangles and a rectangle. The side opposite the 30° angle is 3 making the hypotenuse of the triangle which is a leg of the trapezoid 6. Therefore, the perimeter is 8+14+6+6=34.

C 26. $\frac{\pi^2}{2\pi r}=8, r=16$

A 27. The dimensions of the photo by itself is x by x+3. The dimensions of the frame are x+1 by x-2. Find the area of the frame and subtract the area of the photo and that equals 46. Solving that gives x=11. The original dimensions are 11 by 14.

D 28. Let the sides of the octagon be x which would be the hypotenuse of the triangles. One side of the triangle would be $\frac{12-x}{2}$ AND since the triangles are 45-45-90, the side could also be represented by $\frac{x}{2}\sqrt{2}$. Set these two expressions equal to each other and $x=12\sqrt{2}-12$. x represents the hypotenuse of the triangle but we want the length of a leg. So we must divide the value of x by $\sqrt{2}$. This gives $12-6\sqrt{2}$.

C 29. Let s equal the side of the square so the diagonal is $s\sqrt{2}$. The perimeter of the triangle is 3+4+5 which equals $s\sqrt{2}$. Solving for s gives $6\sqrt{2}$. The area of the square is 72.

B 30. The area of the circle with radius 10 is 100π. The area with the radius decreased by 2 is 64π. The difference of the areas is 36π. The percent decrease would be $\frac{36\pi}{100\pi}=36\%$.

Geometry Team Question #1 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. Find the number of sides of a regular polygon when one exterior angle has a measure of $40°$.

2. Find the degree measure of the largest angle of a quadrilateral whose angle measures are in the ratio of 1:2:3:4.

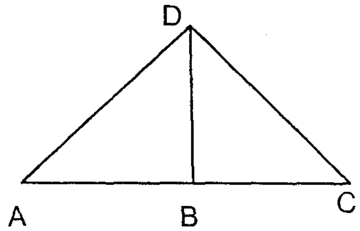
3. Find the length of the median of a trapezoid that has an area of 50 square units and altitude of 10 units.

4. Find the diameter of a circle whose area in square units equals the circumference in units.

Geometry Team Question #2 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

In the diagram, ΔADC is a right triangle with $\overline{DB} \perp \overline{AC}$ and $\overline{AD} \perp \overline{CD}$, $CD = 5$ and $BC = 4$.

1. Find the length of $\overline{AD}$.
2. Find the length of $\overline{AB}$.
3. Find the length of $\overline{BD}$.
4. Find the length of $\overline{AC}$.



Geometry Team Question #3 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. Find the circumference of a circle with endpoints of a diameter at coordinates $(-2,3)$ and $(3,-9)$.

2. Find the area of the smaller of two similar triangles with corresponding perimeters of 2 and 6 and the area of the larger triangle is 81.

3. Find the area of a rhombus that has one diagonal twice the other and a perimeter of $20\sqrt{5}$.

4. Find the number of sides in a convex polygon with 170 diagonals.

Geometry Team Question #4 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. In math class, students were asked to draw a polygon and its diagonals. Ralph drew a triangle, Buddy drew a quadrilateral, Harry drew a pentagon, Curt drew an octagon, and Alan drew a polygon. Curt's polygon had the number of diagonals that was 12 more than one-half the sum of all the other diagonals. Find the number of sides Alan's polygon has.

2. Point P is 13 inches from the center of the circle with radius 5 inches. Find the length in inches of the tangent segment to the circle from point P.

3. A rectangle having a side of length 15 is inscribed in a circle of diameter 17. Find the area of the rectangle.

4. The minute hand of a clock is 6 inches long. Find the number of inches the tip of the hand travels during the time from 10:00 am to 2:45 pm?

Geometry Team Question #5 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. Find the area of a triangle with sides of 9, 16, and 21.

2. Find the area of a triangle with vertices $(3,-4),(-2,5),(-1,6)$.

3. Find the area of ΔABC with $AC = 2\sqrt{2}, m\angle B = 45, m\angle A = 30$.

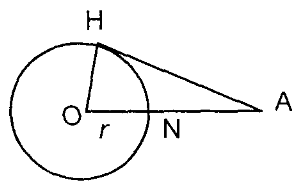
4. Find the sum of the lengths of the diagonals in a rhombus with a sides of 10 and one angle having measure of 60 degrees.

Geometry Team Question #6 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

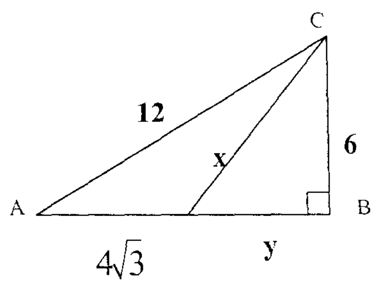
1. A quadrilateral is inscribed in a circle. Two angles measure 50 and 103. Find the measures of the other two angles.

2. A nonagon has 5 congruent angles and 4 other angles which measure 30, 140, 120, and130. Find the measure of the exterior angle of the largest angle.

3. In the figure, O is the center of the circle, $\overline{AH}$ is tangent to the circle at H, $ON = NA$ and the radius of the circle is r. Find, in terms of r, the exact area of the region inside ΔAOH and outside the circle. Express your answer as a single fraction.



4. Determine the value of x in the diagram.

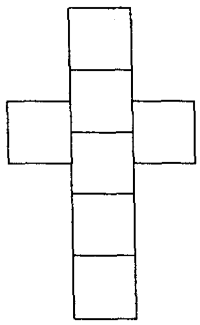


Geometry Team Question #7 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. Find the area between a square and its circumscribed circle when the side of the square is 8.

2. Find the number of sides of a regular polygon with the ratio of the interior angle to exterior angle is 2:1.

3. Find the perimeter of the diagram shown formed by seven congruent squares where the area of the diagram is 343.

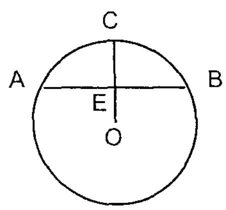


4. Find the perimeter of an equilateral triangle whose area is $16\sqrt{3}$.

Geometry Team Question #8 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. Find the length of $A'B'$ when $\Delta ABC \sim \Delta A'B'C'$, $AC = 8, AB = 7, A'C' = 3$.

2. In the figure below of $\odot O$, the radius $\overline{OC}$ has a measure of one. Find the area of ΔABC in terms of x, where x is the length of $\overline{OE}$.



3. Find the difference in the measures of the acute angles of a right triangle when the square of the hypotenuse is equal to twice the product of the legs

4. Find the longer leg in a 30-60-90 triangle when the median to the hypotenuse has a length of 5.

Geometry Team Question #9 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. Equilateral ΔABC is revolved about side $\overline{BC}$ as an axis of rotation. Find the exact distance traveled by point A in one revolution if BC = 6.

2. ΔABC is isosceles with base $\overline{AC}$, $AB = 10\text{cm}$, and $m\angle ABC = 120°$. Find the length of $\overline{AC}$ in cm.

3. Find the length of the radius of a circle in which a 48 cm chord is 8 cm closer to the center than a 40 cm chord.

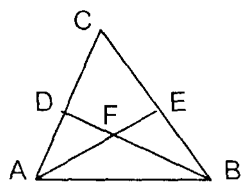
4. The measure of the supplement of an angle exceeds three times the measure of the complement by 10. Find the complement of the angle in degrees.

Geometry Team Question #10 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. Find the slope of the line that passes through the points $\left(\sqrt{2}, \sqrt{3}\right)$ and $\left(\sqrt{18}, 2\sqrt{12}\right)$.

2. Find the perimeter of a triangle formed by connecting the midpoints of the sides of a triangle with sides having lengths 6, 6, and 8.

3. Find the $m\angle AFB$ in ΔABC where altitudes $\overline{AE}$ and $\overline{BD}$ are drawn and F is the point of intersection of the altitudes (see diagram). $m\angle CAB = 80°, m\angle CBA = 60°$



4. Find the length of a leg of an isosceles trapezoid where the bases are 33 and 43 and the distance between the bases is 12.

Geometry Team Question #11 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. In $\Delta FGH, \overline{GJ}$ (with J on $\overline{FH}$) bisects $\angle FGH, FG = 10, GH = 8, FH = 12.6$.
   Find the length of $\overline{FJ}$,

2. Find the length of the smallest side of a right triangle when the perimeter is 30 and the sum of the squares of the 3 sides is 338.

3. The diameter of the front wheel of a tricycle is 10 inches and the diameter of each of the back wheels is 5 inches. How many revolutions has one of the back wheels made when the front wheel has turned $1080°$?

4. Find the distance between the midpoints of $\overline{AB}$ and $\overline{CD}$ where the endpoints of $\overline{AB}$ are $(-12, 7)$ and $(2, 5)$ and the endpoints of $\overline{CD}$ are $(-4, -20)$ and $(10, -4)$.

Geometry Team Question #12 Florida Invitational Middleton TIGERS February 24, 2007
NO CALCULATOR

1. Find the volume of a right circular cone when the diameter of the base is 18 and the slant height is 15.

2. Find the ratio of the diagonal of a square to the diameter of a circle when the figures both have areas of 24.

3. Find the number of sides of a regular polygon with one interior angle having a measure of $156°$.

4. The sides of a triangle are 4, 5, and 6. Each side is trisected and the points of division are joined to form a hexagon. Find the perimeter of the hexagon.

Geometry Team Question #12 **Middleton Invitational February 24, 2007**

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Geometry Team Question #13 **Middleton Invitational February 24, 2007**

1. If a 12 inch chord of a circle is bisected by a 15 inch chord, find the number of inches in the shorter segment of the 15 inch chord.

2. How many posts 12 feet apart in a straight line are needed to hold a wire fence 180 feet long? Each endpoint of the fence has a post.

3. The length of a side of the smaller of two similar triangles is 300 cm while the corresponding side of the larger is 6 m. Find the ratio of the areas of the larger to the smaller.

4. Find the length, in cm, of a side of a square when the diagonal is 10 cm longer than a side.

Geometry Team Question #14 **Middleton Invitational February 24, 2007**

1. Find the length of the diagonal of a rectangular solid with dimensions, 4, 5, 8.

2. Find the area of a triangle with sides having lengths 4, 13, and 15.

3. Find the sum of the measure of the six exterior angles of a triangle.

4. Find the radius of the circle $x^2 + y^2 - 4x + 8y - 5 = 0$.

SOLUTIONS

Question #1

1. 9 2. 144 3. 5 4. 4

1. The exterior angle has measure 40, $\frac{360}{40} = 9$ sides
2. $10x = 360, x = 36$. The $4x$ would be the largest angle so its measure is 144.
3. The area is found by finding the product of the median and the altitude, so the median would be 5.
4. $\pi r^2 = 2\pi r$, solving gives $r = 2$ and the diameter would be 4.

Question #2

1. $\frac{15}{4}$ **2.** $\frac{9}{4}$ **3. 3** **4.** $\frac{25}{4}$

Using Pythagorean triples, $DB = 3$. Using the fact that $\overline{DB}$ is the altitude to the hypotenuse, $DB = \sqrt{BC \bullet AB}; 3 = \sqrt{AB \bullet 4}; AB = \frac{9}{4}$. And using Pythag, $AD^2 = AB^2 + BD^2; AD^2 = \frac{81}{16} + 9; AD = \frac{15}{4}$. $AC = 4 + \frac{9}{4} = \frac{25}{4}$

Question #3

1. 13π **2. 9 3. 100 4. 20**

1. Use the distance formula to find the diameter is 13, making the circumference 13π
2. Since the ratio of the sides is 1:3, the ratio of the areas is 1:9. Set up the proportion $\frac{1}{9} = \frac{x}{81}, x = 9$ which is the area of the smaller triangle.
3. Diagonals in a rhombus are perpendicular so right triangles are formed.
   The side of the rhombus is .$5\sqrt{5}$ To avoid using fractions, let one diagonal be 2x and the other 4x. Diagonals bisect each other so in the right triangle with sides of x, 2x and $5\sqrt{5}$, use Pythag, $x^2 + (2x)^2 = 125, x = 5$. One diagonal is 10 and the other is 20. The area of a rhombus is ½ the product of the diagonals or 100.
4. $\frac{n(n-3)}{2} = 170; n^2 - 3n - 340 = 0; n = 20,$ ~~-17~~

Question #4

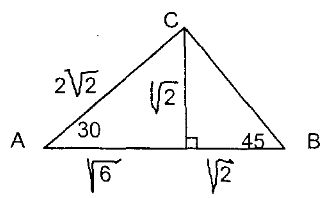
1. 6 **2.** 12 **3.** 120 **4.** 57π

1. The triangle has 0 diags, the quad has 2, the pentagon 5, and octagon 20.
   Let x = the number of diagonal of the polygon. The equation to solve is
   $20 = 12 + \frac{1}{2}(7 + x)$. $x = 9$ which is the number of diagonals of the polygon.
   We want the number of sides so $\frac{n(n-3))}{2} = 9, n = 6$.
2. Once the segment from P to the center is drawn, and the radius to the tangent, we now have a right triangle. We are looking for a leg. Since the radius is 5 (a leg), and the length from P to the center is 13 (the hypotenuse), the other leg is 12.
3. When a rectangle is inscribed in a circle, the diagonal of the rectangle is the diameter of the circle. Since the triangle formed by the diagonal, the length and width of the rectangle is a right triangle, use Pythagorean theorem with 15 and 17 and find the width is 8. The area is 8 times 15 or 120 square units.
4. 10 AM to 2:45 PM is 4 ¾ trips around the circle. So 4 ¾ times the circumference which is 12π gives 57π.

Question #5

1. $14\sqrt{23}$ **2.** 7 **3.** $\sqrt{3}+1$ **4.** $10+10\sqrt{3}$

1. use Heron's Theorem, the semiperimeter is 23, $\sqrt{23 \bullet 2 \bullet 7 \bullet 14} = 14\sqrt{23}$
2. $\begin{array}{cccc} 3 & -2 & -1 & 3 \\ -4 & 5 & 6 & -4 \end{array} = |15 - 12 + 4 - 8 + 5 - 18| = \frac{14}{2} = 7$
3. Use the diagram below with measures as computed. The area of the triangle would be $\frac{1}{2}\sqrt{2}\left(\sqrt{6}+\sqrt{2}\right) = \sqrt{3}+1$.



4. Drawing the diagonals makes 30-60-90 triangles. Since the hypotenuse is 10, the sides opposite the 30 degree angle would be 5 making one diagonal 10, and the side opposite the 60 degree angles would be $5\sqrt{3}$ making the other diagonal $10\sqrt{3}$. The sum would be $10+10\sqrt{3}$.

Question #6

1. 130, 77 **2. 12** **3.** $\frac{3r^2\sqrt{3}-\pi r^2}{6}$ **4.** $4\sqrt{3}$

1. In a quadrilateral inscribed in a circle, opposite angles are supplementary so the supplement of 50 is 130 and the supplement of 103 is 77.
2. The sum of the interior angles of a nonagon is 1260. The sum of the 4 given angles is 420, and the 5 congruent angles have a sum of 1260-420 which is 840. Dividing 840 by 5 gives each of the congruent angles as 168. Since that is the largest angles, its exterior angles has a measure of 12.
3. Since ON=NA, OA has a measure of 2*r*. The radius is *r*. Since the hypotenuse is twice a leg, the triangle is 30-60-90, making HA = $r\sqrt{3}$. To find the area of the region inside the triangle and outside the circle, find the area of ΔOHA and subtract the area of the sector. The area of the triangle is $\frac{1}{2} \bullet r \bullet r\sqrt{3} = \frac{r^2\sqrt{3}}{2}$. The $m\angle HON = 60$ so for the area of the sector, $\frac{60}{360} \bullet \pi r^2 = \frac{\pi r^2}{6}$. Subtract these two and get $\frac{r^2\sqrt{3}}{2} - \frac{\pi r^2}{6} = \frac{3r^2\sqrt{3}-\pi r^2}{6}$.
4. Since ABC is a right triangle and leg BC is ½ the hypotenuse, the triangle is a 30-60-90. AC= $6\sqrt{3}$ making $y = 2\sqrt{3}$. Using Pythag, $x^2 = 36+12, x = 4\sqrt{3}.$

Question #7

1. $32\pi - 64$ **2. 6** **3. 112** **4. 24**

1. The area of the square is 64. The diagonal of the square is $8\sqrt{2}$ which makes the radius of the circle $4\sqrt{2}$ and the area of the circle is 32π so the area needed is $32\pi - 64$
2. $3x = 180, x = 60$ which is the exterior angle which makes the polygon a hexagon which has 6 sides.
3. The area of one square is 49 so the sides are 7. There are 14 sides which comprise the perimeter so 16 x 7 = 112.
4. Area = $16\sqrt{3}$, using 30-60-90 triangle, the side = 8. Perimeter = 3s=24.

Question #8

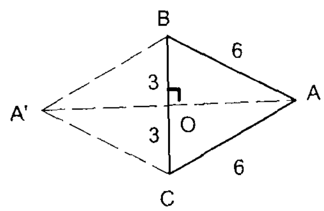
1. $\frac{21}{8}$ **2.** $(1-x)\sqrt{1-x^2}$ **3.** 0 **4.** $5\sqrt{3}$

1. $\frac{7}{x}=\frac{8}{3}, x=\frac{21}{8}$
2. The area of the triangle will be $\frac{1}{2}\bullet AB \bullet CE$. Since $OE = x, CE = 1-x$.
   To find AB, we will find EB using right triangle EOB. Using Pythag, $EB=\sqrt{1-x^2}, AB = 2\sqrt{1-x^2}$. The area of the triangle is $\frac{1}{2}\bullet 2\sqrt{1-x^2}\bullet(1-x)=(1-x)\sqrt{1-x^2}$.
3. $c^2 = 2ab$ and $c^2 = a^2 + b^2$, $2ab = a^2 + b^2$, solving gives $a = b$ so the triangle is isosceles and the angles are 45. The difference between the two acute angles is 0.
4. In a right triangle, the midpoint of the hypotenuse is equidistance from the vertices making the hypotenuse 10, longer leg is opposite the $60°$ angle and has length $5\sqrt{3}$.

Question #9

1. $6\pi\sqrt{3}$ **2.** $10\sqrt{3}$ **3.** 25 **4.** 40

1. Point A will travel the circumference of the circle centered at point O, the midpoint of segment BC. Since BC = 6, AB = AC = 6 and BO = CO = 3. $\overline{AO}$ is the altitude of ΔABC and if BO = 3, then $AO = 3\sqrt{3}$. Point A's path is the circumference of circle O, $C = 2\pi r = 2\pi(3\sqrt{3}) = 6\pi\sqrt{3}$.



2. Draw the altitude from vertex angle B to base AC. That gives two 30-60-90 triangles. The altitude is 5 and each part of the base is $5\sqrt{3}$ so the base AC is $10\sqrt{3}$ cm.
3. For the chord with length 48, draw the segment from the center to the chord. Let this be x. The segment is perpendicular to the chord and bisects it so one part has length 24. Draw the radius from the center to the end of the chord. We now have a right triangle and using Pythag $x^2 + 24^2 = r^2$. Doing the same for the other chord gives the equation

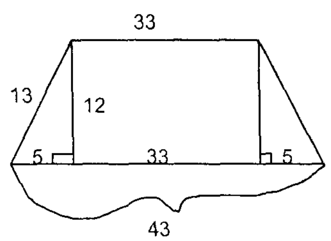
$(x+8)^2+400=r^2$. Solving this system gives $x=7$. Using this and the right triangles makes the radius 25.

4. $180-x=3(90-x)+10, x=50, C=40$

Question #10

1. $\frac{3\sqrt{6}}{4}$ **2. 10** **3. 140** **4. 13**

1. $\frac{3\sqrt{6}}{4}$
2. The perimeter of the triangle formed by connecting midpoints is half the perimeter of the original triangle. 10
3. Using the right triangles, $m\angle ABF=10, m\angle FBE=50, m\angle EAB=30, m\angle AFB=140$
4.



Question #11

1. 7 **2. 5** **3. 6** **4.** $2\sqrt{97}$

1. GJ is an angle bisector so use the triangle angle bisector theorem and set up the proportion $\frac{10}{x}=\frac{8}{12.6-x}, x=7$.
2. $a+b+c=30, a^2+b^2+c^2=338$ and $a^2+b^2=c^2$. Substituting, $2c^2=338, c=13$. The sum of the legs is 17. The legs are 5 and 12. The smallest side measures 5.
3. The front wheel has rotated three times. Therefore, it has traveled a distance of $3(\pi d)$ or 30π. Each back wheel has traveled the same distance. Let x = number of rotations. Then, $30\pi = x(\pi d), 30\pi = x(5\pi), x=6$.
4. The midpoints are $(-5,6)$ and $(3,-12)$. Using the distance formula gives the distance as $2\sqrt{97}$.

Question #12

1. 324π **2.** $\frac{\sqrt{2\pi}}{2}$ **3. 15** **4. 10**

1. The slant height is 15 and radius is 9 making the altitude 12.
   $V = \frac{1}{3}Bh = \frac{1}{3} \bullet 81\pi \bullet 12 = 324\pi$
2. Area of a square an be found by $\frac{d^2}{2}$. Setting this equal to 24 gives
   $\frac{d^2}{2} = 24, d = 4\sqrt{3}$. Area of the circle is $\pi r^2 = 24, r = \frac{2\sqrt{6}}{\sqrt{\pi}} d = \frac{4\sqrt{6}}{\sqrt{\pi}}$.
   $\frac{4\sqrt{3}}{\frac{4\sqrt{6}}{\sqrt{\pi}}} = \frac{4\sqrt{3\pi}}{4\sqrt{6}} = \frac{\sqrt{2\pi}}{2}$.
3. One exterior angle is 24, to find the number of sides, divide 360 by 24 which is 15.
4. Draw the triangle and mark the points of trisection. Draw the hexagon. Put the lengths of the segments on the diagram. Since the 3 triangles formed by drawing the hexagon are congruent, the lengths of the sides of the hexagon are 2, $\frac{4}{3}$, and $\frac{5}{3}$ making the perimeter of the hexagon twice the sum of the lengths of the sides of the triangles or 10.

Question #13

1. 3 **2. 16** **3. 4:1** **4.** $10+10\sqrt{2}$

1. The 12 inch chord is bisected into segments each having a length of 6. The 15 inch chord would have lengths of x and $15-x$. Using a power theorem, $6^2 = x(15-x); x = 12, 3$. The shorter length was asked for so that is 3.
2. 180 divided by 12 is 15. Add one for the end. 16
3. 300 cm is 3 m. The ratio of the sides is 1:2 making the ratios of the areas, larger to smaller, 4:1.
4. Let the side of the square be x. This would make the diagonal $x+10$ and also $x\sqrt{2}$. Set these two expression equal and solving for x. $x = 10+10\sqrt{2}$

Question #14

1. $\sqrt{105}$ **2. 24** **3. 720** **4. 5**

1. Using the formula $\sqrt{l^2+w^2+h^2} = \sqrt{16+25+64} = \sqrt{105}$.
2. Using Heron's formula, $\sqrt{16 \bullet 12 \bullet 3 \bullet 1}$ =24.
3. 720
4. $x^2+y^2-4x+8y-5=0; x^2-4x+4+y^2+8y+16=5+4+16$, r=5.