Palm Harbor University High School
January Invitational Competition 2006
Algebra II Individual

1. If $\log_2(\log_3(\log_4 xy)) = 1$, what is x-y if x is 64 times y?

   A. 16,638 B. 768 C. 4080 D. 4032 E. NOTA

2. If $x + y = 6$ and $x^2 + y^2 = 20$, what is $x^3 + y^3$?

   A. 216 B. 72 C. 125 D. 115 E. NOTA

3. Find the length of the line that connects an endpoint of the latus rectum and vertex of the parabola $y = 2x^2 + 7x - 14$.

   A. $\frac{\sqrt{3}}{8}$ B. $\frac{1}{8}$ C. $\frac{1}{4}$ D. $\frac{\sqrt{5}}{8}$ E. NOTA

4. Find the total number of squares on a 5×5 checkered board.

   A. 53 B. 54 C. 55 D. 56 E. NOTA

5. If $f(x) = (x^2 + 2x)$, and $f(g(x)) = (x^2 + 10x + 24)$, what is $g(x)$?

   A. x+3 B. x-1 C. x+4 D. x+6 E. NOTA

6. What is the percent probability, to the nearest 10^{th}, of choosing at least 3 hippos for an advisory team of seven if you are choosing amongst 5 hippos and 10 humans?

   A. 42.7 B. 32.6 C. 57.3 D. 67.4 E. NOTA

7. If Walter tosses a rubber hippo off of a 62 foot building and it bounces up to 70 percent of its previous height, what is the total vertical distance traveled in feet?

   A. $144\frac{2}{3}$ B. $413\frac{1}{3}$ C. $206\frac{2}{3}$ D. $351\frac{1}{3}$ E. NOTA

8. Find the domain of the inverse of $x = \sqrt{3 - y}$

   A. $[0, \infty)$ B. $(-\infty, 3]$ C. $(-\infty, -3] U [3, \infty)$ D. $(-\infty, 0]$ E. NOTA

9. If Caitie's Car Care can wash 18 cars an hour while Margo's Marvelous Mobile can wash 20 cars an hour if both utilize their entire respective work forces, then about

how many hours will it take a third of Margo's company and half of Caitie's company to wash Donald Trump's 526 cars?

A. 33.6 B. 32.9 C. 27.7 D. 13.8 E. NOTA

10. John's happiness varies directly with the square of the number of hours he sleeps. If his happiness is at 122 HU's (happiness units) when he sleeps 4 hours, then how many hours of sleep did John get the night before his happiness was at 252 HU's, assuming that he is not allowed to sleep during the day?

A. 15.9 B. 4.9 C. 10.6 D. 5.7 E. NOTA

11. Find the sum of the digits of the number 86253_9 when placed in base 3

A. 12 B. 18 C.24 D. 30 E. NOTA

12. If a line crosses the point (-2, 4) and is perpendicular to the line -2x+3y=44, find the x-intercept of it.

A. -8 B. $\frac{2}{3}$ C. $\frac{14}{3}$ D. $\frac{-3}{2}$ E. NOTA

13. $\begin{pmatrix} 3 & 1 & 0 \\ 1 & 5 & -2 \\ -3 & 2 & -1 \end{pmatrix} M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Find the sum of all elements of matrix M.

A.2 B. $\frac{3}{2}$ C. 3 D. 6 E. NOTA

14. Find the coefficient in the fourth term of the expansion of $(3x-7y)^6$.

A. -9,621 B. 59,535 C. -185,220 D. 21,609 E. NOTA

15. If $\log_x 3x+10=2$, solve for x

A. $\{5,-2\}$ B.$\{5\}$ C. $\{-2\}$ D. $\varnothing$ E. NOTA

16. What is the sum of the first 16 positive multiples of 5?

A. 600 B. 640 C. 680 D. 720 E. NOTA

17. How many of the following are odd functions?

I. $y = x^3 + 7x + 5$
II. $y = 3x^4 + 9x^2 + 15$
III. $x = 5y^5 + 7y^3 + 15y$
IV. $y = (x+3)^3$

A. 1 B. 2 C. 3 D. 4 E. NOTA

18. Which of the following is equal to $-i^{38}$?

A. i B. -1 C. $-i$ D. 1 E. NOTA

19. If 14 people are in a room and each person shakes each other person's hand exactly once, then how many handshakes will have occurred in total?

A. 91 B. 105 C. 182 D. 210 E. NOTA

20. Jognog has a 16 ounce can of 45% apple juice, with the rest being water. If he pours 3 more ounces of pure apple juice into the mixture, what is the new concentration of apple juice, rounded to the nearest 10th place as a percentage?

A. 37.9% B. 53.7% C. 63.8% D. 49.7% E. NOTA

21. What is the probability of getting exactly two distinct pairs in a five card poker hand, rounded to the thousandths place?

A. .048 B. .095 C. .103 D..104 E. NOTA

22. Which of the following sets are closed under multiplication?

A. the rational numbers B. the imaginary numbers C. the irrational numbers
D. all of the above E. NOTA

23. Find the area of the conic section: $9x^2 + 4y^2 + 16y - 36x + 16 = 0$

A. 36π B. 4π C. 9π D. 6π E. NOTA

24. $\sum_{i=1}^{10} 3i - 2 = ?$

A.117 B. 140 C.176 D.145 E. NOTA

25. $\frac{4^9}{8^3} = \left(\frac{1}{8}\right)^x$

A. 3 B. -3 C. $\frac{1}{3}$ D. $\frac{-1}{3}$ E. NOTA

26. $\frac{5i+7}{6+3i}$ can be simplified to which of the following?

A. $\frac{9}{15}+\frac{1}{5}i$ B. $\frac{3}{5}+\frac{17}{15}i$ C. $\frac{17}{15}-\frac{3}{5}i$ D. $\frac{19}{15}+\frac{1}{5}i$ E. NOTA

27. $\log_4 8 + \log_9 243 = ?$

A. 2 B. 3 C.4 D.5 E. NOTA

28. $f(x) = x^2 - 3x + 7$, find the remainder when $f(x)$ is divided by $x-3$

A. 7 B. 16 C. 27 D. 5 E. NOTA

29. How many positive integral factors does the number 252 have?

A. 4 B. 6 C. 9 D. 18 E. NOTA

30. What is the maximum value of the function $f(x) = -3x^2 + 4x - 5$?

A. $\frac{2}{3}$ B. $-\frac{2}{3}$ C. $\frac{11}{3}$ D. $-\frac{11}{3}$ E. NOTA

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1. D Working inwards, the $\begin{aligned}\log_3(\log_4 xy) &= 2\\ \log_4 xy &= 9\\ xy &= 4^9\end{aligned}$. The only working combination where x is 64 times greater than y is where $y = 4^3, x = 4^6$. $4^6 - 4^3 = 4032$

2. B If $x + y = 6$, then $(x+y)^2 = x^2 + y^2 + 2xy = 36$. Since $x^2 + y^2 = 20$, $xy = 8$. $(x+y)^3 = x^3 + y^3 + 3xy(x+y) = 216$, so substituting our earlier values back into the equation, $\begin{aligned}x^3 + y^3 + 24(6) &= 216\\ x^3 + y^3 &= 72\end{aligned}$.

3. D The coefficient of the parabola (a) is equal to $\frac{1}{4p}$, where $4p$ is the total length of the latus rectum and p is the distance from the focus to the vertex. Therefore, $\begin{aligned}2 &= \frac{1}{4p}\\ p &= \frac{1}{8}\end{aligned}$, and our triangle formed is right with one side length of $\frac{1}{8}$, another side $\frac{2}{8}$ (since the triangle only includes 2p of the latus rectum, and the hypotenuse x is $(\frac{1}{8})^2 + (\frac{2}{8})^2 = x^2$, with $x = \frac{\sqrt{5}}{8}$.

4. C Finding a pattern, the number of total squares on any given chessboard is the sum of the squares of integers less than or equal to it, up till 0. Therefore, the total number of squares in this case is $5^2 + 4^2 + 3^2 + 2^2 + 1^2 = 55$

5. C $(g(x))^2 + 2(g(x)) = (g(x))(g(x)+2) = (x+4)(x+6)$, so $g(x)$ must be equal to x+4

6. A The total probability will be the sum of the probability of choosing 3, 4, or 5. Each case is then added and divided by the total number of combinations. So,
$$\frac{5C3 \times 10C4 + 5C4 \times 10C3 + 5C5 \times 10C2}{15C7} \approx 42.7\%$$

7. D The distance traveled becomes an infinite series, multiplied by 2 for the "fall". However, since the ball only fell from a 62 foot building but did not rise, we must subtract that amount. Therefore, $2\left(\frac{62}{1-.7}\right) - 62 = 351\frac{1}{3}$ feet.

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8. B By switching x and y to obtain the inverse, our function is $y=\sqrt{3-x}$. Since the quantity $3-x\geq 0$, $x\leq 3$, or in the set $(-\infty,3]$

9. A $\frac{1}{3}$ Margo's rate= $\frac{20}{3}$ cars/hour, $\frac{1}{2}$ Caitie's rate= 9 cars/hour. Since rt=d, $t(r_1+r_2)=d$, so solving 16t=526 yields $t\approx 33.6$ hours.

10. D $h=ks^2$, 122=k(16), so k=$\frac{61}{8}$. With 252 HU's, $252=(\frac{61}{8})s^2$, resulting in an s of about 5.7 hours.

11. C $86253_9=8(9^4)+6(9^3)+2(9^2)+5(9)+3(9^0)$. However, since $3^2=9$, $86253_9=8(3^8)+6(3^6)+2(3^4)+5(3^2)+3(3^0)$, meaning that the digits in the number are the same, with a sum of 24.

12. B The line -2x+3y=44 has a slope of $\frac{2}{3}$ (subtracting and dividing into y=mx+b form). Since a line perpendicular must have a slope that, when multiplied by $\frac{2}{3}$, gives -1, it can be solved and found to be $\frac{-3}{2}$. Put into point slope form, our line is $\frac{-3}{2}(x+2)=y-4$. When y=0 (x-intercept), x= $\frac{2}{3}$

13. E Matrix M fits the definition of an inverse matrix of $\begin{pmatrix} 3 & 1 & 0 \\ 1 & 5 & -2 \\ -3 & 2 & -1 \end{pmatrix}$, which we may call N. Therefore, $M=\begin{pmatrix} 3 & 1 & 0 \\ 1 & 5 & -2 \\ -3 & 2 & -1 \end{pmatrix}^{-1}$. Transposing and replacing with its minors, then multiplying by $(-1)^{i+j}$, where i and j are row and column, respectively, our adjoint matrix is $\begin{pmatrix} -1 & 1 & -2 \\ 7 & -3 & 6 \\ 17 & -9 & 14 \end{pmatrix}$. Now, since the determinant of our original matrix is 4, we must multiply the adjoint by ¼, which gives us our final inverted matrix $M=\begin{pmatrix} -1/4 & 1/4 & -1/2 \\ 7/4 & -3/4 & 3/2 \\ 17/4 & -9/4 & 7/2 \end{pmatrix}$ The sum of the elements in M is $\frac{15}{2}$.

14. C When expanded out, it is found that every term is multiplied by every other term to create the expansion. As the first term involves all 3x's, we want one that has 3 3x's and 3 -7y's. Because multiple combinations yield this, our term will be $(3x)^3(-7y)^3\left(6C3\right)=-185220x^3y^3$, with a coefficient of -185220.

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15. B $\log_x 3x+10=2$, so $x^2-3x-10=0=(x-5)(x+2)$. The solutions are 5 and -2, but negative 2 cannot be the base of a logarithm, so the solution set consists of just 5

16. C $\sum_{i=1}^{16} 5i$, the sum of any arithmetic series can be defined (for S=sum, n=# of terms, d=difference between each term, and a=first term) as $S=\frac{n}{2}\left(2a+(n-1)d\right)$. S=680 in this case.

17. E Odd functions mean that -f(x)=f(-x). Since number III is not a function, and none of the others satisfy the previous conditions, the answer 0, or NOTA.

18. D $i=i, i^2=-1, i^3=-i, i^4=1, i^5=i\ldots$. Therefore, $i^{38}=-1$ and $-i^{38}=1$

19. A Each of the 14 people can shake the other 13s' hands one time, giving you 14(13). However, that would be double the amount actually shaken, because we are assuming that A shaking B is the same as B shaking A. So, dividing by two, the total handshakes are 91.

20. B The amount of pure apple juice Jognog has currently is 7.2 ounces. With 3 added ounces, the total would be 10.2. Since the denominator, or "total liquid" is also increased by 3, the concentration is $\frac{10.2}{19}\approx 53.7\%$

21. A The first pair can be drawn from any of 13 sets of same cards, and only 2 of them need to be picked. The next pair fits the same description only you must choose from 12 sets. The final card can be any card in the deck other than ones that are the same number as the first two pairs, or you would have a full house. We double count this way, though, because we make no distinction between the same two pair of kings and queens in Since the total number of card combinations is 52C5, our probability is $\frac{13\times 4C2\times 12\times 4C2\times 44C1}{52C5\times 2}\approx .048$

22. A Closed means that any elements in the set can be put under the specified operation and yield an element of the original set. The imaginary numbers are not closed $(i\times i=-1)$, and neither are the irrational numbers ($\pi\times\frac{1}{\pi}=1$), but the rational numbers are.

23. D $9x^2+4y^2+16y-36x+16=0$ can be rewritten as $\frac{(x-2)^2}{2^2}+\frac{(y+2)^2}{3^2}=1$, signifying an ellipse with a semi-major and a semi-minor axis of 2 and 3, respectively. The area of an ellipse is πab, where a and b are the semi-major and semi-minor axes. The area of the specified ellipse is then 6π

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24. D $\sum_{i=1}^{10} 3i-2=3\sum_{i=1}^{10} i-\sum_{i=1}^{10} 2=3(5)(11)-2(10)=145$

25. B $\frac{4^9}{8^3}=2^9, \left(\frac{1}{8}\right)=2^{-3}$, meaning $2^9=2^{-3x}$. Taking the $\log_2$ of both sides gives us $9=-3x, x=-3$

26. D $\frac{5i+7}{6+3i}=\frac{(7+5i)(6-3i)}{45}=\frac{19}{15}+\frac{1}{5}i$

27. C $\log_4 8+\log_9 243=\frac{\log 8}{\log 4}+\frac{\log 243}{\log 9}=\frac{3\log 2}{2\log 2}+\frac{5\log 3}{2\log 3}=\frac{8}{2}=4$

28. A By the remainder theorem, the remainder when dividing a polynomial f(x) by a linear factor in the form $(x-k)$ is equal to the f(k). $3^2-3(3)+7=7$

29. D $252=2^2 3^2 7$. Since an integral factor can choose any combination of the amounts of the three factors, including 0, you can have (3)(3)(2)=18 combinations or positive integral factors

30. D The maximum value, in this case, is the x-coordinate of the vertex of the parabola. In vertex form, $f(x)=-3\left(x-\frac{2}{3}\right)^2-\frac{11}{3}$, with $-\frac{11}{3}$ signifying the location of the y-coordinate of the vertex, which is the answer we seek.

Palm Harbor University High School Invitational – 2006 - Algebra II Team #1

1. $f(x) = x^4 - 15x^2 + 10x + 24$ Find the sum of the cube of the roots of f(x).

Palm Harbor University High School Invitational – 2006 - Algebra II Team #2

2. A= the number of ways one can choose 5 socks, 8 shirts, and 3 pairs of underwear from a (messy) drawer of 10 socks, 10 shirts, and 15 pairs of underwear

   B= the probability of rolling at least an 8 when rolling two fair six-sided die simultaneously

   C= the number of distinct permutations of the words PALM HARBOR (assuming the space is fixed)

   D= the number of distinct ways to arrange 7 distinct symmetrical keys on a keychain

   Find: A-5C+D/B

Palm Harbor University High School Invitational – 2006 - Algebra II Team #3

3. Pump A, Pump B, and Pump C all fill up the same pool in 4 hours, 5 hours, and 6 hours, respectively.

   A= the number of hours it takes for pump A and C, on simultaneously, take to fill the given empty pool

   B= the number of hours it takes A and B to fill the empty pool working together

   C= the number of hours it takes B and C to fill the empty pool working together

   D= If the pumps empty at the same rate they fill, the number of hours it takes pump A to empty a filled pool while pump B is filling the same pool.

   Find A+B+C+D

Palm Harbor University High School Invitational – 2006 - Algebra II Team #4

4. Find the fourth term in the binomial expansion of $(x+3)^5$

Palm Harbor University High School Invitational – 2006 - Algebra II Team #5

5. A= $\log_3 243$

   $\log_B 8 = 1.5$

   $4^{C+1} = 5^C$

Palm Harbor University High School Invitational – 2006 - Algebra II Team #6

6. A= the least common multiple of 36, 378, 147

   B= the greatest common factor of 1,936,000, 23,760, and 6,600

   C= the number of zeroes consecutively at the end of 451!

   D= the greatest integral amount by which $2n^3 + 3n^2 + n$ must be divisible, for any positive integer n

   A-B+C-D

Palm Harbor University High School Invitational – 2006 - Algebra II Team #7

7. A= If x is inversely related to y and directly related to the square of z, and x is 4.03919 when y is 7 and z is 3, find y when x is 7 and z is 4.

B= the value of $f(3)$ if $f(2x-3)=x^2+2x$

Find the value of B-A, rounded to the nearest thousandth

Palm Harbor University High School Invitational – 2006 - Algebra II Team #8

8. $\sum_{i=1}^{\infty} \frac{1+i}{2^i}$

Palm Harbor University High School Invitational – 2006 - Algebra II Team #9

9. $\begin{vmatrix} -3 & 2 & 5 & 4 \\ 8 & -1 & -3 & 7 \\ 5 & 3 & -9 & 8 \\ -6 & 6 & -7 & -2 \end{vmatrix}$

Palm Harbor University High School Invitational – 2006 - Algebra II Team #10

10. For the graph $9x^2 + 2y^2 - 36x - 12y + 18 = 0$,

A= the value of the abscissa of the center of the graph
B= the value of the ordinate of the center of the graph
C= the focal distance from the center of the graph
D= the length of the minor axis of the graph

Solve: A-B+CD$\sqrt{14}$

Palm Harbor University High School Invitational – 2006 - Algebra II Team #11

11. Palm Harbor Mu Alpha Theta has \$302 surplus out of their annual budget. (All interest rates are annual)

A= the value of the money compounded monthly with a 7.5% interest rate for 2 years
B= the value of the money compounded quarterly with 6.3% interest for 6 years
C= the maximum value that the money can attain, using compound interest, after 3 years at 4.5% interest.

Give A+B-C to the nearest cent

Palm Harbor University High School Invitational – 2006 - Algebra II Team #12

12. For the equation $3x-4y=18$, A is equal to the x-intercept, and B is equal to the slope of the line perpendicular to the given one.

For the equation $2y+3x=-5$, C is equal to the y-intercept, and D is equal to the area of the polygon the graph makes with the negative x and y axes.

Solve: $\dfrac{\text{B-AC}}{\text{D}}$

Tiebreaker #1

A=the sum of the first 10 Fibonacci numbers

B= the sum of all the numbers in the 9th row of Pascal's triangle

C= $\sum_{n=1}^{26} n^2$

Find C-A-B

Tiebreaker #2

For the parabola $y = ax^2 + bx + c$ with points (3,58), (-5,10), and (-4,-33) on the parabola, find $\frac{a}{b+c}$.

Tiebreaker #3

Farmer Cody knows that any one yam has a 79% chance of growing to maturity; with all the yams in his plot being independent of each other and taking into account all factors. If he plants 14 yams, let A be the probability that exactly 10 of them grow to maturity, B be the probability that half grow to maturity, C be the probability that all grow to maturity and D be the probability that Cody at least 11 grow to maturity.

Find: A+B+C+D, to the nearest ten thousandth.

Palm Harbor University High School Invitational
January 21, 2006
Algebra II Team Solutions

1. $f(x) = x^4 - 15x^2 + 10x + 24 = (x+4)(x+1)(x-2)(x-3)$
The sum of the cubes of the roots= $(-4)^3 + (-1)^3 + 2^3 + 3^3 = -30$

2. A=$10C5 \times 10C8 \times 15C3 = 5{,}159{,}700$
B= There are 5 ways to roll an 8, 4 ways to roll a 9, 3 ways to roll a 10, 2 ways to roll an 11, and 1 way to roll a 12, giving you 15 ways to roll above an 8. The total number of possibilities is (6)(6), so your probability is 5/12
C= There are 10! ways to arrange the letters, divided by the number of ways to arrange the indistinct letters (the 2 A's and the 2 R's), so the net number of permutations is $\frac{10!}{2!2!} = 907{,}200$
D= There are 7! ways to put the keys down on a flat table, but you must divide by 7 because each arrangement has seven indistinct "rotations." That number should be divided by 2 because an arrangement forward is the same as one backward when the keychain is flipped. Therefore, D= $\frac{7!}{(7)(2)} = 360$

A-5C+D/B= $5{,}159{,}700 - (5)(907{,}200) + \frac{360}{5/12} = 624{,}564$

3. Since d=rt and there's only 1 pool, the rate of pump A is ¼ pool per hour, pump B 1/5 pool per hour, and pump C 1/6 pool per hour. So, we can say $\begin{matrix} r_A = 1/4 \\ r_B = 1/5 \\ r_C = 1/6 \end{matrix}$.

A=$1/(r_A + r_C) = \frac{12}{5}$, B=$1/(r_A + r_B) = \frac{20}{9}$, C=$1/(r_B + r_C) = \frac{30}{11}$, D=$1/(r_A - r_B) = 20$

A+B+C+D=$\frac{13538}{495}$ or $27.3\overline{49}$

4. Since the first term has 5 powers of x and 0 powers of 3, the fourth term has 2 powers of x and 3 powers of 3. However, since there are different ways to choose this among the written out expansions, our fourth term is $x^2(3)^3(5C2) = 270x^2$

5. $3^5 = 243$, A=5
$B^{1.5} = 8, B = 4$
$4^{C+1} = (4^{\log_4 5})^C$
$C = \frac{1}{\log_4 5 - 1} \approx 6.213$

A+B-C=2.787

$36 = 2^2 3^2$

6. A- $378 = 2^1 3^3 7^1$ so the lcm is $5292 = 2^2 3^3 7^2$

$147 = 3^1 7^2$

$1936000 = 2^7 5^3 11^2$

B- $23760 = 2^4 3^3 5^1 11^1$, so the gcd is $440 = 2^3 5^1 11^1$

$6600 = 2^3 3^1 5^2 11^1$

C- A 5 matched up with a 2 makes a 10, and, therefore, a 0. Since there are more 2's t han 5's, we just need to find the number of factors of 5. To do this we can divide by 5 repeatedly, yielding integral quotients, until no more integral multiples remain when dealing with factorials. 451/5=90.2. 90.2/5=18.04 18.04/5=3.608. So there shall be 90+18+3=111 0's at the end.

D= $23760 = 2^4 3^3 5^1 11^1$ Either n or n+1 must be divisible by 2, and assuming

$6600 = 2^3 3^1 5^2 11^1$

neither is divisible by 3, n and n+1 must be congruent to either 1 or 2 in mod 3. If one is 1, the other must be 2, which means that $n + n + 1 = 2n + 1 \equiv 3 \equiv 0 (\bmod 3)$, so the number is divisible by at least 2 and 3, or 6.

A-B+C-D=4957

7. A=Defining x, we see $x = \frac{kz^2}{y}$ for some constant k. Substituting, $4.03919 = \frac{k9}{7}$, so $k \approx 3.14159$. Substituting a second time knowing k,

$$7 = \frac{(3.14159)9}{y}$$
$$y \approx 7.181$$

B= In order for this to work, 2x-5=3, or x=4. Substituting, $f(3) = 4^2 + 2(4) = 24$

24-7.181=16.819

8. $\sum_{i=1}^{\infty} \frac{1+i}{2^i} = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} ...\right) + \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} ...\right) + \left(\frac{1}{8} + \frac{1}{16} + \frac{1}{32} ...\right) + ...$ Our final value becomes a sum of a sum. The first term is $\frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$ and our second term is $\frac{\frac{1}{4}}{1 - \frac{1}{2}} = \frac{1}{2}$,

so our new infinite series has first term 1 and common difference $\frac{1}{2}$, so the final sum is $\frac{1}{1-\frac{1}{2}}=2$.

9. $\begin{vmatrix} -3 & 2 & 5 & 4 \\ 8 & -1 & -3 & 7 \\ 5 & 3 & -9 & 8 \\ -6 & 6 & -7 & -2 \end{vmatrix} =$

$-3\begin{vmatrix} -1 & -3 & 7 \\ 3 & -9 & 8 \\ 6 & -7 & -2 \end{vmatrix} - 2\begin{vmatrix} 8 & -3 & 7 \\ 5 & -9 & 8 \\ -6 & -7 & -2 \end{vmatrix} + 5\begin{vmatrix} 8 & -1 & 7 \\ 5 & 3 & 8 \\ -6 & 6 & -2 \end{vmatrix} - 4\begin{vmatrix} 8 & -1 & -3 \\ 5 & 3 & -9 \\ -6 & 6 & -7 \end{vmatrix}$, which, when evaluated in the same way separately, equals -565.

10. The graph, which is an ellipse, can be rearranged as $\frac{(x-2)^2}{2^2}+\frac{(y-3)^2}{(3\sqrt{2})^2}=1$, with a center (2,3), and the length of the semi-major and semi-minor axes $3\sqrt{2}$ and 2, respectively. The focal distance is then $\sqrt{(3\sqrt{2})^2-2^2}=\sqrt{14}$. The minor axis length is 2(2)=4. This means that A-B+CD$\sqrt{14}=2-3+4\cdot(14)=55$

11. $P_f = P_o(1+\frac{r}{n})^{nt}$, A= $A=302\left(1+\frac{.075}{12}\right)^{12(2)} \approx 350.710$

B= $B=302\left(1+\frac{.063}{4}\right)^{4(6)} \approx 439.431$ $\quad P_f = P_0e^{rt}$ for continuously compounded interest, or the maximum value it can attain. C= $P_f = 302e^{.045(3)} \approx 345.650$

$A+B-C \approx \$444.49$

12. $3x-4y=18$ Setting y=0, the x intercept is equal to 6. Solving for y, the slope of the line given is $\frac{3}{4}$, and therefore a perpendicular line has a slope of $-\frac{4}{3}$.

$2y+3x=-5$ Setting x=0, the y intercept is $-\frac{5}{2}$. To find the area, we can solve for the x-intercept of the graph (set y=0, x-intercept is $-\frac{5}{3}$) and, drawing the line, we find it is a right triangle with legs of length $\frac{5}{2}$ and $\frac{5}{3}$. $\frac{1}{2}\left(\frac{5}{2}\right)\left(\frac{5}{3}\right)=\frac{25}{12}$

$$\frac{-\frac{4}{3}-(6)\left(-\frac{5}{2}\right)}{\frac{25}{12}}=\frac{164}{25}\text{ or }6.56$$

Tiebreaker #1

A. $\sum F_n + 1 = F_{n+2}$. 1,1, 2, 3, 5, 8, 13, 21, 34, 55, 89,144 **143**

B. $2^9 = 512$

C. $\frac{(26)(27)(53)}{6} = 6201$

Tiebreaker #2

Setting up a system of equations $\begin{array}{c} a(9)+b(3)-65=58 \\ a(25)+b(-5)-65=10 \\ a(16)+b(-4)-65=-33 \end{array}$, we solve it to yield a=7, b=20, and c= -65. $\frac{7}{20-65}=\frac{7}{45}$

Tiebreaker #3

A=$14C10(.79)^{10}(.21)^4 \approx .18432$

B=$14C7(.79)^7(.21)^7 \approx .01187$

C=$(.79)^{14} \approx .03688$

D=$14C11(.79)^{11}(.21)^3 + 14C12(.79)^{12}(.21)^2 + 14C13(.79)^{13}(.21) + .03688 \approx .66341$

A+B+C+D $\approx .8965$