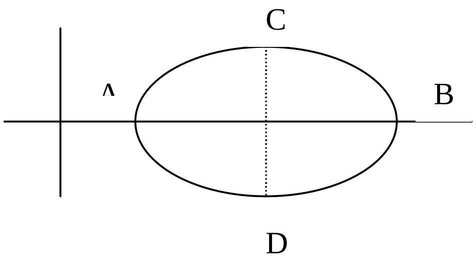
2001-2002 **American Computer Science League** **Contest #4**

Junior Division **Programming Problem**



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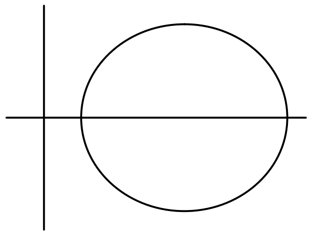

PROBLEM: The figure on the left above is an ellipse and the figure on the right above is a circle. They are related mathematically because the equations used to describe each figure are similar. The **general form** of the equation for both is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. For the circle A and C must be equal and B must equal 0. For the ellipse, A and C cannot be equal but must have the same sign. B must equal 0. An example of an equation of a circle is: $x^2 + y^2 + 4x - 6y - 3 = 0$. An example of an equation of an ellipse is: $x^2 + 4y^2 - 6x - 16y - 11 = 0$. In this program you will be given a general form of the equation and must determine if it is a circle or an ellipse. Further, you will have to give certain other information about the figure.

The general equation above is not very useful for determining information about the position of the figure on the coordinate axis system. The equation must be modified. The method commonly used is called “completing the square”. Working with the equation of the circle given above, first rearrange the terms and leave the spaces as follows:

$$x^2 + 4x + \_\_ \quad + y^2 - 6y + \_\_ \;=\; 3$$

Next, using the “completing the square” method, fill in the two blanks so as to make each trinomial a perfect square trinomial. For the first blank, this is done by dividing the coefficient of the x term by two and then squaring the result. Likewise, dividing the coefficient of the y term by two and then squaring the result fills the second blank. Since two values have been added to the left side of the equation, those same values must be added on the right. The result is as follows:

$$x^2 + 4x + \mathbf{4} + y^2 - 6y + \mathbf{9} = 3 + \mathbf{4} + \mathbf{9}$$

note: ($4 = (4/2)^2$ and $9 = (6/2)^2$)

$$x^2 + 4x + 4 + y^2 - 6y + 9 = \mathbf{16}$$

note: (16 = 3+4+9)

The first three terms are a perfect square trinomial and can be factored as $(x + 2)^2$ and the last three terms, also a perfect square trinomial, can be factored as $(y-3)^2$. Replacing the trinomials with their factored forms gives:

$$(x + 2)^2 + (y - 3)^2 = 16$$

In this form, it is easy to find the center of the circle and it’s radius. The center is at (-2, 3) and the radius is 4 (the principal square root of 16).

Finding information about the ellipse is done in a similar manner. However, since the “completing the square” method only works when the coefficient of the squared term is a positive one, the second trinomial must be modified.

Junior Division Programming Problem

$x^2 + 4y^2 - 6x - 16y - 11 = 0$

$x^2 - 6x + \quad + 4y^2 - 16y \quad = 11$

$x^2 - 6x + \mathbf{9} \quad + \mathbf{4}(y^2 - 4y + \quad) = 11 + \mathbf{9}$

note: ($\mathbf{9} = (6/2)^2$ and the **4** is factored out to make the coefficient of y^2 a 1.

$x^2 - 6x + 9 \quad + \mathbf{4}(y^2 - 4y + \mathbf{4}) = 11 + 9 + \mathbf{16}$

note: ($\mathbf{16} = 4 * 4$) and ($\mathbf{4} = (4/2)^2$)

$(x - 3)^2 + 4(y - 2)^2 = 36$

In this form, the center of the ellipse can be found to be at (3, 2).

If both sides of the equation are divided by the value on the right side, the equation is now said to be in **standard form**:

$$\frac{(x-3)^2}{36} + \frac{(y-2)^2}{9} = 1$$

In the figure of the ellipse above, line segments AB and CD are axes. Since segment AB is longer it is called the major axis and since segment CD is shorter it is called the minor axis. The bigger denominator above determines the major axis. Its length is found by taking two times the principal square root of the denominator. Here the major axis will have a length of 12 and in a similar manner, the minor axis will have a length of 6.

INPUT: There will be 5 sets of data. Each set will contain six integers A, B, C, D, E and F that represent the coefficients of the general equation.

OUTPUT: For each set of data, print whether it is a circle or an ellipse. If it is a circle, print the location of its center in ordered pair format (x,y) and its radius. If it is an ellipse, print the location of its center and the length of its major axis.

SAMPLE INPUT	**SAMPLE OUTPUT**
1. 1, 0, 1, 4, -6, -3	1. Circle, (-2,3), 4
2. 1, 0, 4, -6, -16, -11	2. Ellipse, (3,2), 12
3. 2, 0, 2, 8, 12, -6	3. Circle, (-2,-3), 4