2004 - 2005 **American Computer Science League** **Program #7**

All-Star Contest

ACSL Fields

PROBLEM: An ACSL field is a mathematical system involving the elements of a set over two operations, @ and !, and that has all the properties listed below where @ and ! are some defined operations:

CLOSURE: The results of a @ b and a! b are unique elements of the set.

COMMUTATIVE: a @ b = b @ a and a ! b = b ! a

ASSOCIATIVE: a @ (b @ c) =(a @ b) @ c and a ! (b ! c) =(a ! b) ! c

IDENTITY: There exists an element (x) in the set, such that for every element a :
a @ x = a and a ! x = a

INVERSE: For every element (a) in the set, there exists a unique element (x) in the set such that : a @ x = the identity. Just as in the operation of multiplication under the real numbers where 0 does not have an inverse ($0 \times \frac{1}{0}$ is undefined) in an ACSL field one element for one operation is allowed to not have an inverse.

DISTRIBUTIVE:
For @ over !: a @ (b ! c) = a @ b ! a @ c

The following are the definitions of 2 operations @ and ! on set A = {0,1,2} :

@	**0**	**1**	**2**
0	0	1	2
1	1	2	0
2	2	0	1

!	**0**	**1**	**2**
0	0	0	0
1	0	1	2
2	0	2	1

INPUT: The first input line will give the elements of set A as a string. For the above example the string would be 012. The next line of input will give the table data for operation @ as a string with the data given across the rows. The string for table @ above would be 012120201. The next line of input will give the table data for operation ! as a string, with the data given across the rows. The string for table ! would be 000012021. The last line of input will represent three elements (x, y and z) of set A. Since there are only 3 elements in this example, the elements could be in any order. An example is: 2,1, 0.

OUTPUT: Find the result of the following operations or questions (if a result does not exist print NONE).

1. x @ y 2. x ! z 3. y @ (x @ z) 4. (z ! x) ! y 5. The Identity element for @. 6. The identity element for !. 7. The inverse of x in @. 8. The inverse of z in !. 9. x ! (y @ z) 10. If the defined system is a field, print YES, else print the names of all the properties and the operators that do not hold (ex. COMMUTATIVE FOR !).

SAMPLE INPUT

INPUT LINE 1: 012
INPUT LINE 2: 012120201
INPUT LINE 3: 000012021
INPUT LINE 4: 2, 1, 0

SAMPLE OUTPUT

1. 0
2. 0
3. 0
4. 0
5. 0
6. 1
7. 1
8. NONE
9. 2
10. YES