

6. LAW OF SINES

10 POINTS

PROBLEM: In a right triangle, you can use trigonometry functions such as SIN, COS and TAN to find missing angles and the Pythagorean Theorem to find missing sides. In an oblique triangle (no right angle) the Law of Sines is used to find missing parts:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

If $A = 29.5^\circ$, $a = 12.9$ and $b = 8.7$, then using the formula above gives:

$$\frac{12.9}{\sin 29.5} = \frac{8.7}{\sin B}$$

$$\sin B = (.492423)(8.7) / 12.9$$

$$\sin B = .332099$$

$$B = \sin^{-1} .332099 = 19.4^\circ$$

To find the measure of angle B, take the inverse sine (\sin^{-1} or Arcsin of B). This function is available on all scientific calculators. But wait, you don't have a calculator! A problem with using this simple formula is that the angles (A, B, and C) will be given in degrees and the trigonometry functions built into your computer to calculate these functions use radians. Further, although all languages have a SIN function using radians, the mathematical function used to convert the SIN of an angle back to an angle measure (Arcsin or inverse sine) is not usually defined in computer languages. Usually, the only inverse function provided is Arctan. You need to check your language for the exact syntax of this function (usually atan, atan, or arctan).

Further complicating the finding of missing parts is the fact that the provided information may lead to two entirely different triangles or may lead to no triangle at all. We provide the following as examples:

EXAMPLE 1. Given $C = 55.66^\circ$, $c = 8.94$, and $b = 25.1$. Evaluation produces $\sin B = 2.3184$. Since the SIN of an angle is always between ± 1 inclusive, no triangle exists.

EXAMPLE 2. Given $A = 55.33^\circ$, $a = 22.8$ and $b = 24.9$. Evaluation produces $\sin B = .8982$. This produces an angle of 64° . However, the $\sin(180^\circ - 64^\circ)$ is also $.8982$. There may be two possible triangles. The angles of one possible triangle are $B = 64^\circ$ and $C = 60.67^\circ$. The other possible triangle has $B_1 = 116^\circ$ and $C_1 = 8.67^\circ$.

INPUT: There will be 10 inputs each containing values for a, A, b, B, c, C where the lower case letters represent sides and the upper case letters represent angles in degrees. If any of the inputs are 0, then they are missing values.

OUTPUT: Solve the triangle, that is, find all the missing angles. Answers within ± 1 degree of the listed answer will be considered correct.

GIVEN FORMULAS : $1^\circ = .01745329$ radians = $\pi/180$

$$\text{ARCSIN}(X) = \text{ARCTAN}(X / \sqrt{-X * X + 1})$$

The answer is an angle measured in radians

SAMPLE INPUT

1. 12.9, 29.5, 8.7, 0, 0, 0
2. 0, 0, 25.1, 0, 8.94, 55.66
3. 22.8, 55.33, 24.9, 0, 0, 0

SAMPLE OUTPUT

1. B = 19.4, C = 131.1
2. Not a triangle
3. B = 64, C = 60.67, B₁ = 116, C₁ = 8.67