FLORIDA MATHEMATICS LEAGUE

P.O. Box 140507, Gainesville, Florida 32614-0507

All official participants must take this contest at the same time.

Contest Number 2 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **November 20, 2007**

Name ______________________ **Teacher** __________ **Grade Level** ____ **Score** ____

Time Limit: 30 minutes NEXT CONTEST: DEC. 18, 2007 *Answer Column*

2-1. If $(x-10)(x+10) = 0$, what is the value of $(x-1)(x+1)$? — 2-1.

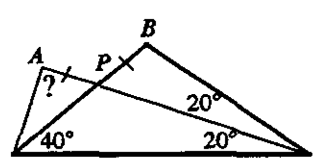
2-2. If the least common multiple of the first 2006 positive integers is m, and the least common multiple of the first 2007 positive integers is km, what is the value of k? — 2-2.

2-3. If 2100 words fill any page that a 23-page local newspaper typesets with large type, and 2800 words fill any page that it typesets with small type, how many pages must the paper typeset with small type so that an article of 56 000 words will exactly fill all 23 pages of some future issue? — 2-3.

2-4. The diagonals of convex quadrilateral Q are perpendicular. If three consecutive sides of Q have respective lengths 3, 9, and 19, then how long is the fourth side? — 2-4.

2-5. I wrote a sequence of n integers. In this sequence, the sum of any 3 consecutive terms is positive, while the sum of any 4 consecutive terms is negative. What is the largest possible value of n? — 2-5.

2-6. The degree-measure of each base angle of an isosceles triangle is 40. The bisector of one of the base angles is extended through point P on the leg opposite that base angle so that $PA = PB$, as shown. What is $m\angle A$? — 2-6.



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Problem 2-1

Since $0 = (x-10)(x+10) = x^2-100$, $x^2 = 100$. The value of $(x-1)(x+1) = x^2-1$ is $100-1 = \boxed{99}$.

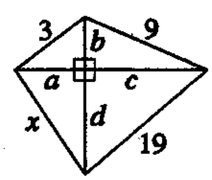
Problem 2-2

Notice that $2007 = 3^2 \times 223$ factors into integers that are already factors of the least common multiple of all the positive integers smaller than 2007. Therefore the least common multiple of the first 2006 positive integers $= m =$ the least common multiple of the first 2007 positive integers $= km$, so $k = \boxed{1}$.

Problem 2-3

If x is the number of pages typeset with large type and y is the number of pages typeset with small type, then $2100x+2800y = 56000$, so $3x+4y = 80$. We also know that $x+y = 23$. Solving, $x = 12$ and $y = \boxed{11}$.

Problem 2-4

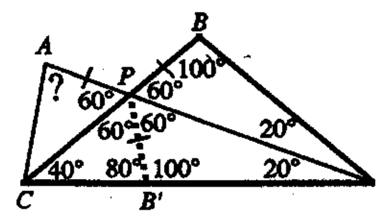


Let's use the Pythagorean Theorem in each of the four right triangles seen in the diagram. We get $3^2+19^2 = a^2+b^2+c^2+d^2$ and $9^2+x^2 = b^2+c^2+a^2+d^2$. Therefore, $3^2+19^2 = 9^2+x^2$, so $x = \boxed{17}$.

Problem 2-5

Some 5-term sequences are −2, −3, 6, −2, −3; or −2, −2, 5, −2, −2; or −3, −3, 7, −3, −3; or −3, −3, 8, −3, −3; or −7, −4, 12, −6, −5. Suppose that c_1, c_2, c_3, c_4, c_5, c_6 is a 6-term sequence in which the sum of any 3 consecutive terms is positive. Then $c_1+c_2+c_3 > 0$, $c_2+c_3+c_4 > 0$, $c_3+c_4+c_5 > 0$, and $c_4+c_5+c_6 > 0$. Adding these 4 inequalities, and rearranging the terms into groups of 4 consecutive terms, we get $(c_1+c_2+c_3+c_4)+(c_2+c_3+c_4+c_5)+(c_3+c_4+c_5+c_6) > 0$. The inequality in the previous sentence cannot be true if the sum of any 4 consecutive terms is negative, so we conclude that there's no such 6-term sequence. Thus, the largest possible value of n is $\boxed{5}$.

Problem 2-6



In the diagram at the right, reflect B to B', reflecting across the line through A and P as shown. This creates 2 new triangles. $\triangle CPA \cong \triangle CPB'$ by SAS, so $m\angle A = m\angle PB'C = \boxed{80 \text{ or } 80°}$.

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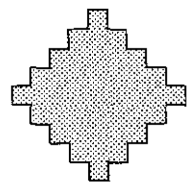
Contest Number 2

Any calculator without a QWERTY keyboard is allowed. Answers must be exact *or* have 4 (or more) significant digits, correctly rounded.

November 18, 2008

Name ____________________ **Teacher** __________ **Grade Level** _____ **Score** ____

Time Limit: 30 minutes NEXT CONTEST: DEC. 16, 2008

	Answer Column
2-1. What is the only negative integer x for which $\|x+2\| = \|x+4\|$?	2-1.
2-2. Into how many unit squares can we partition the 36-sided equilateral polygon shown if its perimeter is 36 and if each of its sides is perpendicular to both sides adjacent to it? 	2-2.
2-3. I have 10 nickels, 10 dimes, and 10 quarters. In how many different ways can I pay for a 45¢ item? (Note: Two ways are different if and only if the number of nickels, dimes, or quarters used is different.)	2-3.
2-4. I cut a roll of red tape into 251 pieces, and then I cut each of the 251 pieces into 8 smaller pieces. *At most* how many cuts did I need to make to produce the resulting 2008 pieces? 	2-4.
2-5. If exactly two different linear functions, f and g, satisfy $f(f(x)) = g(g(x)) = 4x+3$, what is the product of $f(1)$ and $g(1)$?	2-5.
2-6. Two triangles which are *not* congruent can actually have five pairs of congruent parts! For example, triangles with side-lengths 8, 12, 18 and 12, 18, 27 have five pairs of congruent parts (two sides and three angles). If two non-congruent *right triangles* have five pairs of congruent parts, what is the ratio of the length of the hypotenuse of either triangle to the length of that triangle's shorter leg?	2-6.

Contest # 2 — *Answers & Solutions* — 11/20/07

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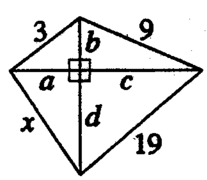

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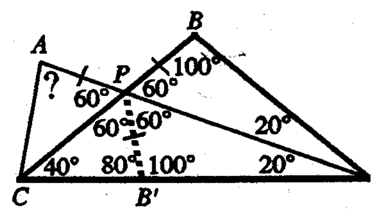

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